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THE RED NEON LINES

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In the course of comparing solar and laboratory wave lengths with the well determined neon standards, some hundreds of plates of the neon spectrum will eventually be measured at the Allegheny Observatory, in cooperation with the Bureau of Standards. The neon tube is of the type described in Bureau of Standards Bulletin, *4*, page 511. The electrodes are aluminum disks 25 mm in diameter, the capillary is 10 cm long with a bore of 2.5 mm. The source of the neon lines measured at Allegheny is a tube which was compared directly with the fundamental standard in an investigation reported in Scientific Papers of the Bureau of Standards, No. 441, *18*, pages 188, 9; 1922. The exciting current was 5000 V ac without capacity, about 50 ma being used. The tube was observed, as always, exactly side on. The A. O. wave lengths published in Tables 1 and 2 were derived by comparison with B. S. values of the neon lines 5852 to 6598 inclusive, and so have no bearing on the ratio of the neon lines to the fundamental standard. The thickness of the interferometer was determined from the mean of all these lines, then the wave length of each line was found from its measured order, in the usual manner. To date 68 plates have been measured which show from 16 to 20 of the stronger lines of neon in the region 5800-6800A. The interferometers used were of 3.75, 6, 8, 10, and 20 millimeters thickness. It seems worth while to collect the results of these measurements in order to study the behavior of the individual lines as the order of interference is increased, and to see if any improvement can be made in the relative values of the wave lengths as published in B. S. Bull., *14*, pp. 765-75; 1918.

In publishing these values, the Bureau of Standards spectroscopists were unwilling to certify the wave lengths to four places of decimals,

as it was possible that some of the lines were in error, relative to the fundamental standard, by a part in six millions, or .001A. The wave lengths in each series of determinations were given to four places whenever the agreement of the different plates seemed to warrant this procedure, since the values of the neon lines were found to have a high degree of accuracy relative to each other. For the present investigation, unweighted means have been taken of the three series of B. S. wave lengths which cover the region. These were all compared directly with the fundamental standard, for the most part by simultaneous exposure. In the case of the lines not occurring in series 2, account was taken of the slight systematic difference which this series shows with respect to the other two. This method of deriving the means leads to the same thousandth angstrom as that adopted by the Bureau of Standards excepting in the case of the weak line 6217. The present investigation shows that the decimal part of this wave length should be .281 to three places, which means giving somewhat greater weight to series 1 than was formerly done (See Table 4).

Perard has reported, in C. R. *176*, p. 375; 1923, that in certain sources the neon lines are double, or even more complex. Buisson and Jausseran find, C. R. *180*, p. 505; 1925, that in some sources the neon lines are reversed, in certain cases with unequal components. Childs, Nature *115*, p. 572; 1925, lists several neon lines which are easily reversed. The line 6402 is cited as a good test object for a 10 mm interferometer, as the line can just be seen as reversed with an order of 30 000. Under the conditions of excitation used at this Observatory, this line is perfectly sharp at an order of 60 000. These facts suggest that considerable care is required in specifying the conditions of operating a neon lamp which is to serve as a source of standard wave lengths. If a line is composed of very close components which are unequal, but do not differ very greatly in intensity, the wave length as determined by means of the interferometer will vary with the order of interference. The values will usually progress with the order, as in the case of 6678 in Table 1, part 1. This type of progression is often found in the case of iron lines which are known to be close doubles, but the behavior of neon 6678 (Table 1) is very likely due to the fact that this line is sometimes outside the field of good definition of the Allegheny spectroscope. The values for this line in Table 1, part 2, do not show this progression. No other line in Table 1 shows this tendency, so it is concluded that the complexity of these lines cause no errors of measurement greater than a few ten thousandths of an angstrom. It is to be noted that the complexities

TABLE 1. *Red neon wave lengths*

	First A. O. Series				Second A. O. Series.				
Thickness	3.75	6	8	20	3.75	6	8	10	20
No. Obsns.	10	7	6	6	9	6	12	8	4
5852.4880	.4862	.4867	.4878	.4873	.4874	.4879	.4867	.4862	.4873
5881.8954	8951	8954	8950	8949	8953	8938	8952	8948	8950
5944.8343	8342	8339	8332	8340	8347	8336	8344	8345	8339
5975.5340	5345	—	5350	5344	5342	5346	5346	5346	5332
6029.9972	9964	9966	9967	9962	9963	9975	9970	9973	9972
6074.3377	3378	3379	3380	3372	3382	3376	3374	3377	3374
6096.1630	1629	1625	1628	1630	1640	1629	1628	1629	1632
6143.0624	0629	0613	0621	0624	0617	0628	0621	0623	0618
6163.5944	5927	5943	5934	5938	5927	5925	5943	5942	5940
6217.2807	2795	2801	2826	2817	2834	2819	2818	2820	2813
6266.4950	4949	4961	4953	4947	4944	4954	4954	4946	4954
6304.7893	7884	7884	7909	7891	7921	7890	7890	7892	7896
6334.4280	4285	4284	4268	4278	4271	4280	4275	4275	4277
6382.9913	9920	9907	9911	9908	9921	9902	9915	9922	9906
6402.2455	2469	2455	2443	2464	2457	2454	2456	2458	2456
6506.5278	5274	5276	5275	5277	5274	5267	5279	5277	5278
6532.8827	8796	8824	8825	8826	8839	8818	8830	8834	8829
6598.9528	9539	9529	9537	9531	9524	—	9516	9522	9542
6678.2760	2736	2748	2752	2765	2758	—	2762	2763	—
6717.0427	—	—	—	0424	0426	—	—	—	—
A. O.−B. S.	± 9	± 5	± 7	± 4	± 8	± 6	± 4	± 5	± 5
do$\times\sqrt{N-1}$	±27	±13	±15	± 9	±22	+13	+13	+13	± 9

mentioned have not been found in the type of source used by the Bureau of Standards and the Allegheny Observatory, using orders up to 100 000.

In Table 1, the first column contains the mean B. S. wave length, derived as explained heretofore. Columns 2, 3, 4, and 5, contain the measurements of the neon lines as made at Allegheny Observatory by means of interferometers of thickness 3.75, 6, 8, and 20 millimeters respectively. Line 2 shows the number of observations by each interferometer. The two bottom lines show the mean difference A. O. minus B. S. for the tabulated means, and this quantity multiplied by $\sqrt{N}-1$.

A second group of observations is similarly represented in Table 1, part 2. A different pair of interferometer plates was used, which fact insures complete independence of the two series of observations.

An examination of the differences A. O. minus B. S., shows that they are inversely proportional to the square root of the thickness of the interferometer: or, the weight is proportional to the order of interference. This rule will apply only to measurements made in the same source and region. In comparing different sources, or the same source in widely different regions, one usually encounters discrepancies which are large compared with the errors of measurement, and which are more nearly the same for all orders of interference, making it wise to weight the results from thin interferometers somewhat higher than has just been indicated.

In Table 2, the first column indicates the intensity of the line, the second column repeats the B. S. wave lengths of Table 1, the next columns give the mean values as resulting from the two A. O. series, and the fifth column contains the mean of all the A. O. determinations for each wave length. The differences A. O. (all) minus B. S. are found in column six. Results by Meissner,[1] Priest,[2] and Wallerath[3] are found in the seventh, ninth and eleventh columns. The bottom line gives the mean of the differences Obs. minus B. S. in ten thousandths angstrom.

Since the means of 29 plates, A. O. series 1, agree with the B. S. values as well as do the means from 68 A. O. plates, it is probable that the accuracy of the B. S. values is of the order of $\pm$. 0003A. The A. O. value of 5852 is unreliable since this line is too close to the edge of the field of the camera. The wave length of the weak line 6217 is not well determined; perhaps the correct value lies between A. O. and B. S. values.

[1] Ann. d. Phys, *51*, p. 115; 1916 and Phys. Zeitschr., *17*, p. 549; 1916.
[2] Bull. Bur. Stds., *6*, p. 573; 1911.
[3] Ann. d. Phys., *75*, p. 37; 1924.

TABLE 2. *Comparison of B. S. wave lengths with other determinations*

Int.	B. S. Mean λ	1st A. O.	2nd A. O.	Mean A. O.	A−BS	Meissner	M−BS	Priest	P−BS	Wallerath	
10	5852.4880	.4870	.4870	.4870	−10	.488	± 0	.4862	−18	488	± 0
6	5881.8954	8949	8950	8950	− 4	896	+ 6	8958	+ 8	894	−14
8	5944.8343	8339	8343	8341	− 2	834	− 3	8344	+ 1	834	− 3
4	5975.5340	5345	5344	5344	+ 4	534	0			534	± 0
4	6029.9972	9964	9970	9967	− 5	999	+18			998	+ 8
7	6074.3377	3376	3376	3376	− 1	337	− 7	3383	+ 6	337	− 7
8	6096.1630	1628	1631	1630	0	163	0	1608	−22	163	± 0
9	6143.0624	0623	0621	0622	− 2	061	−14	0600	−24	062	− 4
5	6163.5944	5935	5934	5934	−10	594	− 4			594	− 4
4	6217.2807	2808	2821	2816	+ 9	279	−17			278	−27
7	6266.4950	4952	4952	4952	+ 2	495	0	4948	− 2	495	± 0
4	6304.7893	7890	7898	7894	+ 1	788	−13	7929	+36	789	− 3
8	6334.4280	4280	4275	4277	− 3	428	0			428	± 0
8	6382.9913	9912	9914	9913	0	991	− 3	9882	−31	990	−13
10	6402.2455	2457	2456	2456	+ 1	246	+ 5	2392	−63		
9	6506.5278	5276	5275	5275	− 3	527	− 8			528	+ 2
4	6532.8827	8828	8823	8825	− 2	881	−17			882	− 7
5	6598.9528	9530	9527	9526	0	953	+ 2			953	+ 2
8	6678.2760	2758	2757	2757	− 1	275	−10			276	± 0
5	6717.0427	0425	——	0425	− 2	042	− 7			042	− 7
Mean	Obs.−B.S.	± 3	± 4	± 3	± 3		± 7		± 21		± 5

The discrepancy in the case of 6163 arises from giving the results of the less reliable thin interferometers too great weight in the Allegheny means. An investigation of the equal wave number differences shows that the B. S. value is essentially correct with respect to the other neon lines, while the Allegheny value is found to be .0007A in error in the direction indicated by the difference A. O. minus B. S. It would therefore be unwise to "correct" the B. S. value by means of the A. O. In a similar manner, the A. O. value of 6217 is shown to be too large, and the B. S. adopted value, .280, too small.

The lines of the neon spectrum have been arranged in series by Paschen, Ann. d. Phys, *60*, pp. 405-53; 1919. Four *s*-terms and ten *p*-terms combine to form the wave numbers of the strong lines in the region 5400-8100A. These terms, computed from the B. S. values of the wave lengths, are shown in Table 3. Assuming $2s_2$, $2s_5$ was computed

TABLE 3. *Term values*

$2s_2$ 38821.1000	$2p_1$ 21739.2734	$2p_6$ 24393.9493
$2s_3$ 39891.1771	$2p_2$ 23671.3566	$2p_7$ 24588.2171
$2s_4$ 40250.5320	$2p_3$ 23792.3781	$2p_8$ 24885.5888
$2s_5$ 40667.9798	$2p_4$ 23851.3011	$2p_9$ 25052.7678
	$2p_5$ 23937.6972	$2p_{10}$ 26452.025−

by means of three pairs of lines. Five pairs were available for computing $2s_5-2s_4$, and three pairs for computing $2s_3-2s_2$. There was only one well observed line each for $2p_3$ and $2p_9$. Two lines could be used for $2p_1$ and $2p_8$, four for $2p_2$, and three for each of the other *p*-terms. Only slightly different values for these terms result from the use of the wave lengths by Meissner or by Wallerath. (If it is necessary to compute the vacuum waves per cm corresponding to several determinations of the wave length of a line in air, it is helpful to use the formula $dn=-n^2d\lambda/10^{10}$. Thus, the value of the vacuum λλ per cm corresponding to 5881.8954 is 16996.6232; for λ5881.8950, $d\lambda=-.0004$. The square of the wave number is 2.89, so $dn=+.0012$, giving 16996.6244 for the vacuum wave number corresponding to .8950).

In Table 4, the second column contains the waves per cm corresponding to the observed B. S. wave lengths. The third column gives the decimal part of the waves per cm derived by subtracting a *p*-term from an *s*-term as indicated in the last column using the data of Table 3. Thus, the wave number of the line 5400 is found by the formula $2s_4-2p_1$. Column 4 shows the wave lengths which are computed from the

data of column 3. Then follow the differences, observed minus computed, for both wave number and λ.

TABLE 4. *Observed and computed wave lengths*

B. S. λ	Waves per Cm		Comp. λ	Obs.−Comp.				$2s-2p$	
	Obs.	Comp.		Wave No.		λ			
5400.5620	18511.4588	4586	5621	+.0002		−.0001		4	1
5852.4880	17082.0264	0266	4879	−	2	+	1	2	1
5881.8954	16996.6232	6232	8954		0		0	5	2
5944.8343	16816.6782	6787	8342	−	4	+	1	5	4
5975.5340	16730.2824	2826	5339	−	2	+	1	5	5
6029.9972	16579.1751	1754	9970	+	3	−	2	4	2
6074.3377	16458.1539	1539						4	3
6096.1630	16399.2313	2309	1631	+	4	−	1	4	4
6143.0624	16274.0316	0305	0629	+	11	−	5	5	6
6163.5944	16219.8206	8205	5943	+	1	+	1	3	2
6217.2807	16079.7633	7627	2809	+	6	−	2	5	7
6266.4950	15953.4809	4779	4954	−	10	+	4	3	5
6304.7893	15856.5824	5827	7892	−	3	+	1	4	6
6334.4280	15782.3904	3910	4278	−	6	+	2	5	8
6382.9913	15662.3152	3149	9915	+	3	−	2	4	7
6402.2455	15615.2122	2122						5	9
6506.5278	15364.9438	9432	5281	−	6	+	3	4	8
6532.8827	15302.9590	9600	8823	−	10	+	4	3	7
6598.9528	15149.7435	7434	9529	+	1	−	1	2	2
6678.2760	14969.7990	7989	2760	+	1		0	2	4
6717.0427	14883.4030	4028	0428	+	2	−	1	2	5
6929.4678	14427.1500	1507	4675	−	7	+	3	2	6

The measurements made at Allegheny and those of Meissner and Wallerath, were similarly treated. The high degree of relative accuracy obtainable in the measurement of lines due to this source is indicated by the small mean values of Obs.−Comp. for the different lists, as shown below:

B. S.	A. O.	Meissner	Wallerath
±.0002A	±.0003A	±.0003A	±.0005A

This procedure readily shows the relative accuracy of different series of measurements of the same group of lines, although the real errors are somewhat larger than is indicated by these mean residuals, since the discrepancies are to a certain extent masked by the method used in computing the values of the terms.

A group of wave lengths showing recurring separations may be examined for internal consistency by the simple process of finding the departure from equality of those separations which are so nearly the same that it is to be presumed that they would be identical under ideal conditions. In this case the errors of two lines are involved in each value of the separation, and the difference in accuracy in different lists of measurements is more readily seen than in the procedure just outlined. The separation which recurs most frequently in this part of the neon spectrum is 417; referring to Table 3 this is seen to be $2s_5-2s_4$. The individual separations are the differences between the wave numbers of two lines whose formulas are $2s_5-2p_i$ and $2s_4-2p_i$, $i=2, 4, 6, 7, 8$. In Table 5 the values of this separation are given in detail: For each of the B. S. series, and for the mean of the three; for the A. O. mean; and for the values of Meissner, Priest, and Wallerath. Altogether, 26 values for the various separations were used to determine the mean departure from equality for each series of observations and the results are shown at the bottom of Table 5.

TABLE 5. $2s_5-2s_4$.

	B.S.1	BS2	BS3	BS	AO	Meissner	Priest	Wallerath
5881−6029	417.4487	—	482	481	479	514	—	543
5944−6096	463	468	473	469	475	478	407	477
6143−6304	495	487	491	492	500	496	645	496
6217−6382	464	487	491	481	454	519	—	519
6334−6506	449	482	463	466	466	448	—	472
Mean	.4472	.4481	.4480	.4478	.4475	.4491	.4526	.4501
	±.0016	07	10	08	12	22	119	24
Mean 26	±.0009	11	12	06	10	14		24

On the basis of internal agreement, any one of the B. S. series is about as good as the mean of all the A. O. plates, and somewhat better than Meissner's or Wallerath's list. Since the mean B. S. has three times the weight of one series, there seems to be no reason for giving either of the other series, more than one third weight in attempting to adjust this mean. Following this procedure, the A. O. results would indicate a change amounting to .0002A in only three cases, and Meissner's results would change the mean by .0004 in four instances. While a change of .0004 does not increase the discrepancies by amounts greater than the present range, a change of .0010A in the B. S. values causes a

noticeable increase in the size of the residuals of the equal separations in which the line is involved.

In the column headed "AO," Table 5, it is seen that the largest value results from subtracting the wave number of a weak line from that of a strong one; the smallest value results from subtracting the wave numbers of a strong line from that of a weak one. This shows an intensity equation which is plainly in evidence in a few of the plates from which the mean was formed. Equal separations furnish a means of checking observations for this and other systematic errors, even if no other series of measurements is at hand. In the present instance, a comparison with the B. S. values shows the intensity equation, but does not tell whether the error is all in one list or partly in each. The values of Priest furnish an example of a pronounced intensity equation. The differences, Priest minus B. S. for intensity 4, 6, 7, 8, 9, and 10, are respectively +.0036, +.0004, +.0002, −.0017, −.0022, and −.0042. While aware that the disappearance method was subject to this error, Priest did not succeed in eliminating it entirely. In the case of the A. O. work, it is reassuring to find this effect present in only a few instances, with the coefficient always small.

In Table 6 the lines are arranged according to terms and some of the separations are tabulated. It is seen that the wave numbers of all the lines under discussion have the formula $2s_i - 2p_j$, four s-terms and ten p-terms being represented. The s-separations in the top line are from Table 3, the columns give the values determined from individual pairs of lines. The first column shows the p-term which characterizes each line; the numbers in column 2 are the decimal parts of the p-separations from Table 3. Additional separations which were used in computing the mean departure from equality are as follows: $2p_5 - 2p_2$; $2p_7 - 2p_2$; $2s_5 - 2s_2$; $2s_5 - 2s_3$. A parenthesis indicates that the inclosed separation was not used in estimating the accuracy of the wave lengths. One of the lines involved in the separation is weak and poorly determined, or it lies outside the region observed at Allegheny. The wave lengths of these weak lines can be computed with great accuracy by means of Table 3.

On the whole, there appears to be little to be gained by changing the adopted B. S. values otherwise than by taking the means to four places. The best reason for making a change is the expectation that individual lines will thus be brought closer to the correct ratio to the fundamental standard, even if the relative values of the neon lines are at the same time made less accurate. The prime consideration is that all series of

wave lengths which are averaged to form secondary standards shall have been compared with the fundamental with the same care which was exercised in the B. S. comparisons. The B. S. means rest on about 50 direct comparisons with the fundamental standard, using orders up to 120 000. These means were checked in another investigation[4] in which some 20 comparisons of neon and the red cadmium were made with high orders. Meissner and Wallerath used a vacuum cadmium arc, and the latter used a type of neon lamp which differs materially from the one which was the source of the lines measured at the Bureau of Standards. It is reassuring to find the mean results of all observers so nearly in accord. Still, it would seem scarcely wise to average measurements of different types of source, either of the neon or cadmium, in order to arrive at precise values of secondary standards. In any case, the internal accuracy of each series should be investigated by means of the equal separations with a view to determining the weight to which each list is entitled.

It is a pleasure to express my indebtedness to my former colleagues in this work, Messrs. Meggers and Merrill, for important suggestions and assistance in connection with this paper.

CONCLUSIONS

As shown by internal agreement, and by comparison with other determinations, the Bureau of Standards values of the red neon lines are sufficiently accurate to justify taking the means to four places of decimals.

No appreciable change in the relative values of the B. S. means is indicated by the results from 68 plates measured at the Allegheny Observatory. (The value 6217.2807 results from giving the three B. S. series equal weight, and this value is preferable to that which has been accepted, viz. 6217.280).

The lines in question have no complexities which interfere with the accuracy of measurement with orders up to 100 000 under the observing conditions described herein.

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[4] Sci. Pap. Bur. Stds., No. 441, *18*, pp. 188, 9; 1922.

TABLE 6. *Recurring separations, B. S. wave lengths.*

Term	$2s_2$		$2s_3$		$2s_4$		$2s_5$
$2s_i-2s_j$ $2p_i-2p_j$	1070.0771		359.3549		417.4478		
$2p_1$	5852				5400		
	17082.2264				18511.4588		
	1932.2832	1932.2829			1932.2836		
$2p_2$	6598		6163		6029		5881
	15149.7435	1070.0771	16219.8206	359.3545	16579.1752	417.4481	16996.6232
	121.0215	(121.03)			121.0213		
$2p_3$	6652				6074		
	15028.71—				16458.1539		
	58.9230	(58.91)			58.9226		
$2p_4$	6678				6096		5944
	14969.7990				16399.2313	417.4469	16816.6782
	86.3962	86.3960			(86.431)		86.3958
$2p_5$	6717		6265		6128		5975
	14883.4030	1070.0779	15953.4809	(359.319)	16312.800–	(417.482)	16730.2824
	456.2521	456.2530			(456.218)		456.2508
$2p_6$	6929				6304		6143
	14427.1500				15856.5824	417.4494	16274.0316
	194.2678	(194.265)			194.2672		194.2683
$2p_7$	7024		6532		6382		6217
	14232.885–	(1070.074)	15302.9590	359.3562	15662.3152	417.4481	16079.7633
	297.3717	(297.376)			297.3714		297.3729
$2p_8$	7173				6506		6334
	13935.509				15364.9438	417.4466	15782.3904
	167.2782						167.2782
$2p_9$							6402
							15615.2122
	1399.215						1399.215
$2p_{10}$	8082		7438		7245		7032
	12369.09—	(1070.06)	13439.149–	(359.358)	13798.507–	(417.450)	14215.957–

RELATIVE GLARE OF MODERATELY COLORED LIGHTS

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The question as to the relative glare of colored lights has frequently arisen. Data on which a correct answer can be based is of special interest since colored headlights are being used upon automobiles regardless of the large absorption of light in the colored lamp bulbs or headlight lenses. The answer will naturally depend upon the basis of comparison; for example the result will depend upon whether their relative glare is compared on the basis of equal candle-power of the actual source (lamp-filament) or the comparison is based on equal illumination at the eye of the observer.

The authors have recently carried out an investigation to determine the relative glare of light from tungsten filaments, after passing through canary, amber, light-blue and colorless glasses, as measured by the reduction in visibility of nearby test-objects. The light-blue glass was similar to but somewhat bluer than that used for bulbs in the "daylight" lamps and blue-bulb automobile lamps. The light through this glass appears bluish in comparison with the light from clear-bulb tungsten lamps, but actually is practically a true white. The "colorless" light was that from a clear-bulb tungsten lamp and actually is an unsaturated yellow. All sources of light were tungsten-filament lamps operating at approximately the temperature of automobile headlight lamps. These lights were tints, the amber being the most saturated color used. The various lights have been compared on the basis of the intensity of illumination reaching the observer's eyes. A description of these investigations with the results obtained, is given below.

No. 1.—This investigation was devised to produce in the laboratory certain conditions comparable with those encountered by an automobile driver when passing other cars on a country highway. The blinding-effect of the approaching headlights is probably near its maximum when the cars are about 75 feet apart, and the glare is experienced for only a short time; hence the work was conducted in such a manner as to expose the observer's eyes to glare sources equivalent to 2900,1100 and 300 candlepower headlights of the different colors at a distance of 75

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feet. These exposures were for periods of nine seconds each, occurring twice a minute, comparable to passing other cars at these intervals. The brightness of the test-object to be discriminated was of the order of magnitude of a dark gray object illuminated by a driver's own headlights.

The arrangement of apparatus employed in this test is shown in Fig. 1. The test-object consisted of an annular opening of 7.5 minutes radial width cut in a thin metal end of box B the interior of which was of brightness F_2. The end of the box, which served as background for the test-object, was painted gray and was illuminated to a brightness of

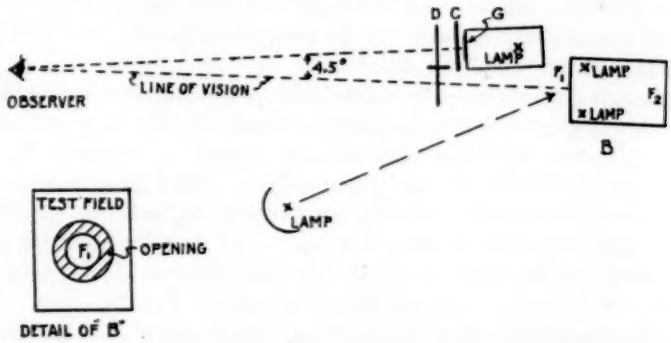


FIG. 1. *Apparatus used in first experiment. Glare source G (a uniformly bright diffusing glass), seen through glass C, and the test field (see detail of B) were intermittently exposed for two periods of 9 seconds in each minute. Observer adjusted voltage of lamps in B until the open ring in the test field just disappeared.*

F_1 millilamberts. The glare-source consisted of a glass diffuser covering a 3.8 cm opening in one end of a cylindrical metal enclosure in which a stereopticon lamp was adjustably placed. A sectored-disk with two 54-degree openings was placed in front of the test-object and glare-source and was rotated at such a speed as to expose simultaneously the test-object and glare-source every 30 seconds. The glare-source was placed 4.5 degrees to the left of the line of vision, and between it and the observer's eye the colored or colorless glass was placed. With the brightness F_1 of the background of the test-object maintained constant, and the glare-source intensity constant, the test consisted in adjusting the brightness F_2 of the test-object so that it was just invisible. The results of this investigation are shown in Table 1.

TABLE 1. *Showing the minimum perceptible brightness-contrasts of test-object and background for various intensities of illumination at the eye for a glare-source located 4.5 degrees to the left of the line of vision and exposed through colored and colorless glasses for 9 seconds, twice a minute. The table shows the average results for five observers. The test arrangements were as shown in Fig. 1. The intensity of illumination E was measured at the observer's eye with the glass interposed.*

Interposed Glass		Ratio* of the brightnesses (F_2/F_1) of test-object and background for certain values of E and F_1.			
Color of Glass	Transmission factor of glass	E=5.6 m.c. F_1=0.066 ml.	E=2.1 m.c. F_1=0.031ml.	E=0.58 m.c. F_1=0.011 ml.	Weighted Average (Relative)
Clear	.90	1.387	1.466	1.571	1.000
Canary	.79	1.381	1.490	1.681	1.020
Amber	.49	1.403	1.486	1.716	1.031
Light Blue	.35	1.376	1.441	1.648	1.003

*Probable error of each ratio = ± .006

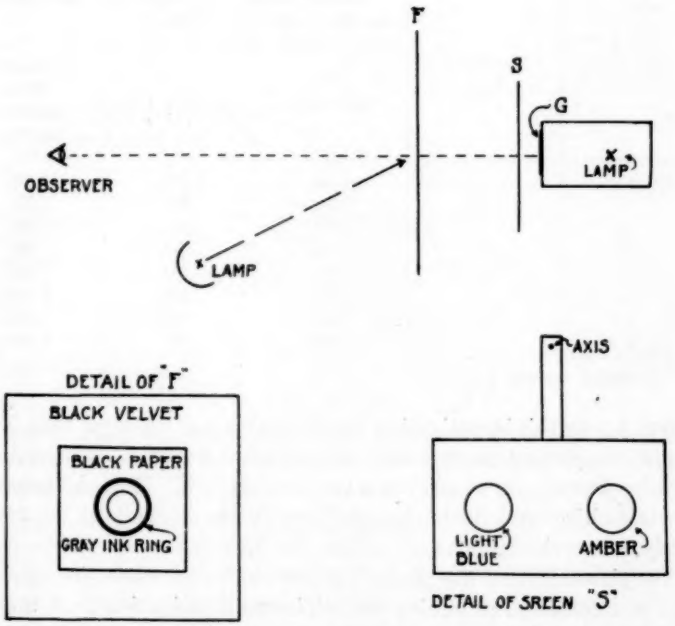


FIG. 2. *Arrangement of apparatus in second experiment. Observer moved screen S until the glare source, seen through the opening in F, was exposed through light-blue or amber glass, and adjusted voltage of lamp behind G until the gray ring on F just disappeared.*

No. 2.—The apparatus was arranged as shown in Fig. 2. The screen *S*, with openings covered with amber and light-blue glass, was under control of the observer and could be moved so as to obscure the glare-source or to place either glass between it and the observer. The concentric gray ring *R*, drawn on black paper, was maintained at a constant brightness. The observer moved the screen *S* about the fixed axis until the glare-source was fully exposed, and while looking directly at the latter he adjusted its voltage until the ring *R* just disappeared. The glasses were alternated, and a short rest-period was introduced between observations. A complete test consisted of ten observations for each color, and in some cases the test was repeated on other days. The results are shown in Table 2.

TABLE 2. *Illumination at observer's eye (with amber or light-blue glass interposed) for equal brightnesses of halo at 0.97 degrees from the line of vision (as determined by disappearance of ring R) when the eyes were focused upon the glare-source. Test arrangement is shown in Fig. 2.*

Observer	Number of Tests	Average illumination at eyes in meter-candles for equal brightness of halo		
		Amber glass	Light-blue glass	Average* ratio, amber to light blue.
No. 1	3	0.512	0.494	1.034
2	3	.492	.441	1.115
3	2	.564	.59	0.961
4	2	.274	.275	0.995
5	1	.817	.756	1.081
6	1	.582	.495	1.174
7	2	.545	.505	1.082
8	1	.258	.253	1.021

* Weighted average $1.053 \pm .015$.

No. 3.—In this investigation the apparatus was the same as in No. 2, Fig. 2, except that the test-field was as shown in Fig. 3. It consisted of a white paper (1 cm square) on a gray background. The brightnesses of the test-object and the background were 0.246 ml. and 0.2 ml. respectively, hence the brightness-contrast was 1.23 to 1.

The glare-source *G* was placed 3 degrees to the left of the line of vision. The test consisted in finding the minimum illumination *E* at the eye, with the colored-glass screen (amber or light-blue) interposed, at which the test-object became just invisible when the observer exposed the glare-source. The latter was exposed for a sufficient period to make a

temporary judgement, which was checked by one or more repetitions. A complete observation consisted of ten such judgements with each color, and was repeated on another day. The results of this investigation are given in Table 3.

TABLE 3. *Showing the meter-candles of illumination at the observer's eyes from a light-source of uniform brightness (3.8 cm in diameter at a distance of 352 cm and 3 degrees to the left of the line of vision) in order to just obscure a test-object 1 cm square and subtending a visual angle of 12.3 minutes. Test-object brightness was 0.246 ml and the background brightness was 0.2 ml. The test arrangement is shown in Fig. 3. Data below are the averages of two sets of observations by each observer.*

Observer No.	Average minimum illumination at eyes for disappearance of test-object		Average** Ratio of Amber to Blue
	Amber glass	Light-blue glass	
1	2.61	2.48	1.061
2	5.07	4.44	1.127
3	5.05	4.04	1.254
4	2.32	2.05	1.129
5*	2.62	2.37	1.138
7	2.65	2.49	1.072
8	6.1	5.12	1.191

* Field brightness reduced 50%.
** Average ratio = 1.139 ± .024.

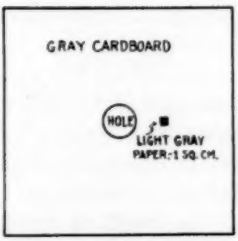


FIG. 3. *Test field in third experiment, instead of field F (Fig. 2). Observer directed his attention at the small square of light gray paper and adjusted brightness of G until the former just disappeared.*

CONCLUSIONS

(1) In investigation No. 1 where the glare-source and test-object were exposed for two nine-second intervals in each minute, the minimum perceptible brightness-contrast between test-object and background was very nearly the same for all the glare-sources tested. It was found to be least for the colorless and not more than 3 per cent greater for

any colored glare-source tested. This difference has no real significance and is practically within the limit of experimental error.

(2) In investigations Nos. 2 and 3, where the length of exposure was under the control of the observer, so that a judgement could be formed at one exposure, the results indicate that for equal visibilities of the test-object an amber glare-source may be about 10 per cent greater in luminous intensity than a glare-source covered with a light-blue glass. In the latter case the actual color of the glare-source is approximately a true white.

(3) Colored light in a headlight is obtained by the use of reflecting or transmitting colored media. These usually absorb about 25 to 50 per cent of the light, and the road illumination will be reduced by this amount unless the wattage of the head-lamps is increased. Hence, the use of such colored devices in headlights would not appear to be justifiable unless it resulted in an improvement in visibility of objects on the roadway when there is no approaching automobile, or else a reduction in the glare experienced by anyone facing the colored headlights. Our investigations indicate that the use of colored headlights (even unsaturated colors or tints) does not result in an appreciable improvement in certain glare and visibility conditions on the road. Inasmuch as our investigations reveal no appreciable advantage for any one of the four unsaturated colors, it is very likely that the use of such colored media for headlights results in a net loss to vision.

DISCUSSION

We appreciate the advantage which a yellowish light may have over a white or bluer one in penetrating power in certain atmospheric conditions, but such conditions are relatively rare and of secondary importance from the viewpoint of automobiling. Certainly, there is little justification for the use of the blue-bulb headlight lamps and there are a number of reasons for concluding that the use of these is disadvantageous. It is well known that colored lights differ markedly in their value for long-range visibility or penetrating power such as is the case in signalling. However, these differences practically disappear in short-range vision such as is the case in automobiling. Furthermore, the latter becomes complicated by glare, by the necessity of seeing illuminated details, etc.

One of the authors (M. L.) concluded some years ago that quantity of light in automobiling is overwhelmingly important compared with quality of light. In general, the facts of physics and of vision favor a

yellowish light as compared with a "white" or seemingly bluish one for automobiling. Then what is the reason for the present extensive use of the blue-bulb headlight lamp? The same author has been inclined to believe that the answer is to be found in the realm of illusion and submits the following as an explanation or at least as an example of the type of explanation to be sought for: Owing to dimmers and to depleted or worn-out batteries, the voltage of lamps is often reduced, and consequently, the yellowishness of the light is increased and the quantity of light is decreased. Through many such experiences *yellowishness* becomes associated with *less light* and *"poorer seeing."* Conversely, when the battery is new or fully charged, when the generator is working well, and when there are no bad contacts, the voltage of the lamps is high and, consequently, the light is less yellowish (that is, whiter) and the quantity of light is greater. In such cases, *whiteness* becomes associated with more light and *"better seeing."* Thus certain associations become established and greatly influence our impressions and judgements without our being conscious of this influence. This is a factor which is certainly involved in any favorable impression of the white light for automobile driving and might adequately explain an impression which is contrary to so many scientific facts.

The powerfulness of associations and illusions may be emphasized by citing the tremendous number of visual experiences which are at variance with the reality. We meet such psychological factors very commonly in our observations; in fact, they are involved in every visual act. In light and vision we are often obliged to go far afield from the realities of physics and of vision to explain impressions and experiences which are seemingly contrary to scientific facts. Our work has naturally taken us often into psychophysiological realms and we have become very respectful of the powers of illusions, associations, etc. Most persons seldom suspect the powers of these influences and, consequently, when they deal with light and vision they are likely to think only in terms of realities. The present popularity of the blue-bulb headlight lamps may well serve as a case for the study of illusions and false impressions. Perhaps the foregoing is not a full explanation or even a partial one. but at least it serves to warn us that impressions are often at variance with realities. The blue-bulb lamp provides a whiteness (the light is not actually blue) which may be unconsciously associated with full voltage with the resulting impression as discussed in the foregoing. If the bulbs are made of a denser blue-glass, a point is likely to be reached where the blueness oversteps itself and the illusion or false impression vanishes

and the driver would say that it was too blue even though the candlepower had been maintained constant. In other words, it is always interesting to press an illusion to the point where it fails to fool us. Our experience indicates that this can be done in the case of blue-bulb lamps.

LIGHTING RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,
NELA PARK, CLEVELAND.

Currents between metal Electrodes across Gaps 20-800 mμ wide. Continuation of Rohmann's earlier experiments, confirming the earlier result that when a current flows across such a gap and is varied either by varying the battery or the resistance in series with the gap, the voltage across the gap varies comparatively little; furthermore, if the width of the gap is varied, the voltage varies more or less proportionally, so that there seems to be a more or less constant "critical field-strength" of the order of a million volts/cm. These particular experiments were performed with 9 different metals, including such extremely different ones as Pt on the one hand, Mg and Ca on the other. They were carried out in a vacuum probably of the order of 10^{-3} mm, with all sorts of vapors probably present. It seems curious that there was no sign of contact-potential difference when dissimilar metals were used on opposite sides of the gap, nor any clear evidence that the "critical field-strength" varies from one pair of metals to another, while on the other hand this critical field-strength varies considerably with the pressure and nature of the gas just previously pumped out, and with the length of time the electrodes are left to stand. Possibly this goes to show that under such conditions as these the character of the "metal" surfaces is determined by the environing gas. Work of other authors has been done under much more rigorous conditions.—[H. Rohmann, Münden; ZS. f. Phys. *31*, pp. 311-325; 1925].

KARL K. DARROW

ON THE RELATION BETWEEN TIME AND INTENSITY IN PHOTOGRAPHIC EXPOSURE

[Second Paper]

By L. A. Jones and E. Huse

INTRODUCTION

In a communication[1] published some months ago in this journal we presented the results of our investigation of the so-called reciprocity law on three well known photographic materials. The data published at that time were the results of an extensive series of measurements and afforded reliable information relative to the relation between the time and intensity factors of exposure. The materials used in this work were Seed 23 plates, Seed 30 plates, and Motion Picture Positive film. From the results with these materials some very definite conclusions were drawn. It was found that (a) there was a definite failure of the reciprocity law, (b) the Schwarzschild equation, $E=I\cdot t^p$, would not fit the observed values, and (c) the experimental determination of the value of "optimal" intensity was subject to great uncertainty due to the flatness of a large portion of the D-log I curves. Furthermore it was found, in the case of one of the materials studied, that gamma (γ) was to a certain extent dependent upon intensity, while in the case of the other two materials γ was independent of intensity.

Since the publication of the paper mentioned, work has been continued in this laboratory on the reciprocity relation and in the present communication some further results are presented A very careful redetermination of the relation between time and intensity for the Seed 30 plate was made and the results are given in the following pages. In addition a critical study has been made of two other photographic materials, Seed Graflex plate and Motion Picture Negative film.

In our previous paper will be found an historical résumé of the literature and a detailed discussion of the experimental procedure It seems unnecessary to repeat these parts at the present time and only in those cases where a modification in experimental technique has been made will this subject be dealt with. For details of apparatus and experimental procedure the reader is referred to the previous communication.

[1] Jones, L. A. and Huse, E. Relation between time and intensity in photographic exposure. J.O.S.A. 7, p. 1079; 1923.

LIGHT SOURCE

Incandescent electric lamps were used for the exposure of the photographic materials. These lamps were carefully seasoned and the voltage required for operation at a color temperature of 2500°K was determined by comparison with temperature standards obtained from the Research Laboratory of the National Electric Lamp Association. The lamps chosen for this work were of the concentrated filament (flood light) type. A tabulation of data regarding these lamps is given in Table 1. Lamp

TABLE 1.

Lamps	Watts	Volts	Effective Candle Power
247	100	74.5	2.50
248	250	85.8	74.20
250	500	80.3	159.70

No. 247 was used in a lamp house provided with a diffusing window made of three sheets of fine ground white optical glass. The "effective candle power" shown in the table is the value obtained using this lamp house and the incandescent lamp as a unit. This accounts for the extremely low value of candle power for the wattage consumption indicated. The lamps No. 248 and 250 were used without diffusing elements between the source and photographic material. The question of selective absorption of the diffusing materials used has been considered further and a large number of measurements made. No indication whatever of selective absorption of sufficient magnitude to be measurable by photographic methods had been obtained. The use of this diffusing material as a means of reducing the intensity, therefore, seems to be entirely justified and free from objection.

PHOTOGRAPHIC MATERIALS

Since it is necessary to make measurements at a large number of different intensity levels and, since at each level a relatively large number of strips must be exposed, the accumulation of data on this subject is rather a slow and laborious process. It seemed desirable, therefore, to limit the number of materials studied at one time. Those chosen for the present work are Seed Graflex plates, Motion Picture Negative film, and Seed 30 plates.

DEVELOPMENT

The elimination of variations in density, arising from variations due to development, has received further consideration. In the work re-

ported in the previous paper considerable difficulty was encountered in obtaining precise repeatability due largely to variations in the reducing power of different mixings of developer. Since this work must necessarily extend over a considerable period of time it is impossible to develop all strips in a single mixing of developer and it was found that various batches of solution made up according to the same formula but at different times gave somewhat different results. After careful consideration it was decided to adopt a new developer for this work. The number of components was reduced to a minimum and chemicals of the highest possible purity were used. In Table 2 is given the formula

TABLE 2. *Standard Developer for Reciprocity Work*

Sodium Carbonate	20 grams
Sodium Sulphite	25 grams
Potassium Bromide	0.5 grams
Pyro	5 grams
Water	1 liter

for the developer adopted. A supply of the necessary chemicals was obtained and the quantities indicated in the formula were weighed out. The carbonate and bromide were placed together in one bottle, the sulphite in another bottle, and the pyro in a third. They were then sealed with paraffin. A large number of these groups were made up previous to starting the work and stored away for future use. When a developer was desired for use one bottle of each of the components was opened and the contents added to one litre of distilled water. The carbonate and bromide were dissolved first, the sulphite next, and then just previous to the starting of development, the pyro was added. This method has proved very satisfactory. As judged by the values obtained, the repeatability over two or three months of work was excellent. This modification in the development technique represents practically the only change from the procedure reported in the previous paper.

RESULTS OBTAINED WITH SEED GRAFLEX PLATES

In Table 3 is given a complete record of exposure conditions for this material. In the first column the number of strips exposed at the various intensity levels is given. The exposed strips were developed under standard conditions as described previously and after fixation, washing and drying under conditions which were also standardized, the densities obtained were determined on the high intensity densitometer.[2] The

[2] Jones, Loyd A. An instrument (Densitometer) for the measurement of high photographic densities. J.O.S.A. 7, p. 231; 1923.

values for all of the strips exposed at a given intensity level were averaged and in Table 4 these density values are given. In the last column of this table are the γ values obtained by plotting these density values as a function of the logarithmic exposure and measuring the

TABLE 3. *Seed Graflex*

Strips	Source	Lamp House	Aperture	Eff. C.P.	D(m)	I(m.c.)	T(max)	I.t (max)
11	250	1		160	.999	160.	.125″	20
12	"	"		"	1.99	40.	.5″	"
11	"	"		"	3.99	10.	2″	"
11	"	"		"	7.99	2.5	8″	"
12	248	"		74.2	1.93	20.	1″	"
12	"	"		"	3.85	5.	4″	"
8	"	"		"	7.70	1.25	16″	"
8	247	2	1	2.5	1.41	1.25	16″	"
8	"	"	1	"	2.83	.3125	64″	"
2	"	"	1	"	5.66	.078	4′−16″	"
12	"	"	2	1.26	8.03	.0195	17′−4″	"
12	"	"	4	.33	8.19	.0049	1^h−8′−16″	"
10	"	"	6	.078	8.02	.0012	4^h−33′−4″	"
11	"	"	8	.019	7.84	.0003	18^h−12′−16″	"

slope of the straight line portion. It will be noted that the variation in γ values for the various groups of strips is practically negligible. Moreover there is no apparent dependence of γ upon intensity. The characteristic curves plotted from the values shown in Table 4 were then

TABLE 4. *Seed Graflex*

log I	Density values for Step Nos.										Gamma
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	
2.2	1.53	1.39	1.34	1.19	1.02	.83	.64	.46	.20	.11	.72
1.6	1.61	1.51	1.39	1.29	1.13	.94	.72	.50	.34	.22	.69
1.3	1.77	1.66	1.54	1.43	1.25	1.05	.83	.58	.38	.23	.71
1.0	1.61	1.50	1.41	1.28	1.12	.95	.71	.47	.32	.19	.70
0.7	1.72	1.60	1.48	1.37	1.23	1.09	.89	.63	.42	.21	.70
0.4	1.78	1.67	1.57	1.47	1.32	1.14	.88	.61	.37	.24	.73
0.1	1.71	1.63	1.54	1.42	1.29	1.14	.95	.70	.40	.23	.71
1.5	1.75	1.68	1.58	1.48	1.31	1.13	.86	.57	.36	.20	.72
2.9	1.74	1.61	1.53	1.41	1.27	1.09	.79	.52	.37	.21	.72
2.3	1.58	1.47	1.37	1.29	1.17	.97	.76	.52	.35	.21	.76
3.7	1.79	1.67	1.59	1.48	1.36	1.14	.87	.60	.39	.25	.73
3.1	1.42	1.35	1.27	1.12	.98	.80	.63	.45	.29	.17	.71
4.5	1.40	1.32	1.22	1.07	.93	.73	.54	.36	.25	.17	.72

corrected for γ and general shape in the manner described in detail in the previous paper. Since for one of the materials previously studied, it was found that γ was to a certain extent dependent upon intensity, it was considered advisable to make specific tests in the case of this material. In Fig. 1 are shown the characteristic curves for three

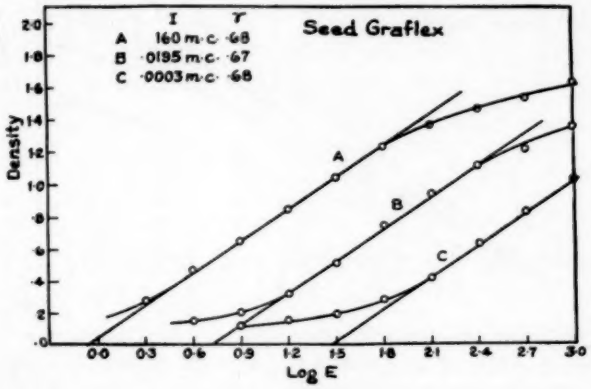


FIG. 1

widely different values of intensity, the exposure time at each intensity being adjusted so as to give approximately equal exposures in the three cases. It will be noted that there is no measurable difference in γ for this material, therefore it must be concluded that γ is independent of intensity. Since this is true it is only necessary to take the arithmetical mean of the γ values shown in the last column of Table 4 as a basis on which to correct the γ values of the various curves. In Fig. 2 the horizontal straight line represents the arithmetical mean,

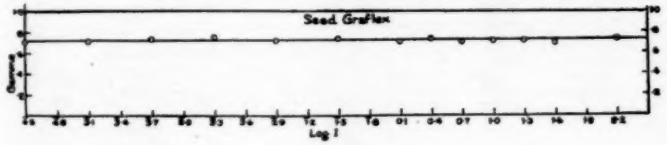


FIG. 2

while the circles show the values of γ obtained at the various intensity levels. This is definite and convincing evidence that γ is not a function of intensity.

Another necessary step in smoothing out possible experimental errors is the determination of the intersection point of a series of exposed

strips developed for different times. It is about this intersection point as an axis that the correction of γ is made. The position of this point is adequately shown in Fig. 3. The value of D at the intersection point

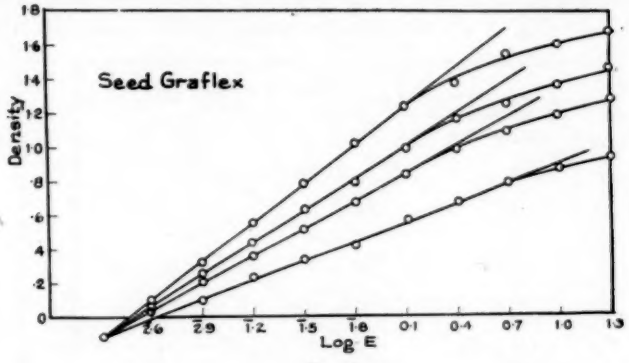


FIG. 3

in this case is $-.11$. Fig. 4 shows the average amount of correction necessary after the determination of the intersection point and average γ, the dotted line representing the corrected curve. The toe and shoulder of the corrected curves were determined by graphical averag-

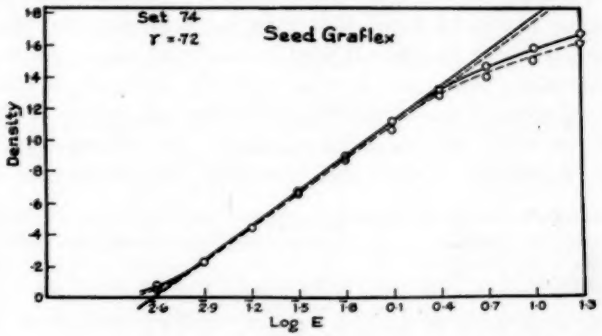


FIG. 4

ing of shape throughout the entire series of curves. The corrected characteristic curve for each group of strips was drawn and from them the density values contained in Table 5 were read. The numbers at the top of each column are the values of $I \cdot t$ to which the indicated densities apply. It will be noted that the first column is for an $I \cdot t$ value

TABLE 5. *Seed Graflex*

log I	Densities for values of $I \cdot t$									
	20	10	5	2.5	1.25	.625	.3125	.156	.079	.039
2.2	1.59	1.50	1.40	1.26	1.06	.84	.62	.41	.20	.07
1.6	1.59	1.50	1.40	1.26	1.06	.85	.63	.42	.21	.09
1.3	1.58	1.50	1.40	1.26	1.06	.85	.63	.42	.21	.10
1.0	1.58	1.50	1.40	1.26	1.06	.85	.63	.42	.22	.10
0.7	1.58	1.50	1.40	1.27	1.06	.85	.63	.42	.22	.10
0.4	1.57	1.50	1.40	1.26	1.06	.85	.63	.42	.22	.10
0.1	1.57	1.49	1.40	1.26	1.06	.85	.63	.42	.22	.10
1.5	1.56	1.48	1.39	1.26	1.05	.84	.62	.42	.21	.10
2.9	1.54	1.47	1.38	1.24	1.04	.83	.61	.41	.21	.10
2.3	1.52	1.45	1.36	1.22	1.02	.82	.60	.40	.20	.09
2.0	1.50	1.44	1.35	1.21	1.01	.81	.59	.39	.19	.08
3.7	1.49	1.42	1.34	1.20	1.00	.80	.58	.38	.18	.08
3.4	1.48	1.41	1.32	1.18	.99	.78	.57	.37	.17	.07
3.1	1.46	1.40	1.30	1.17	.98	.77	.55	.36	.16	.06
4.5	1.42	1.36	1.26	1.14	.94	.74	.52	.32	.13	.04

of 20. From these data for each value of $I \cdot t$ the curves shown in Fig. 5 were plotted. These curves represent density as a function of log I for various constant values of $I \cdot t$. The value of $I \cdot t$ for the top curve of this group is 20, while the value for each succeeding curve decreases from this by a factor of two. In drawing these curves, a final smoothing

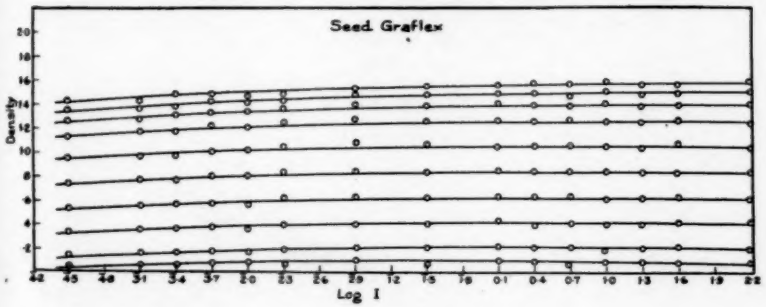


FIG. 5

out of the experimental variations is obtained. In case there were no failure of the reciprocity law the curves shown in this figure would be straight lines parallel to the log intensity axis. It should be noted that there is a perceptible curvature to these lines although the departure from a straight line is not very great considering the enormous range

of intensities included. While there is undoubtedly an intensity which gives a maximum density for constant $I \cdot t$ the location of the position by graphical methods of this so-called "optimal" intensity must be subject to considerable uncertainty.

RESULTS OBTAINED WITH SEED 30 PLATES

The method of reducing the data to final form for this material was similar to that used for Seed Graflex. Although Seed 30 plates were used in the previous paper it was felt that further data on this material would be desirable. A tabulation of the various exposure conditions

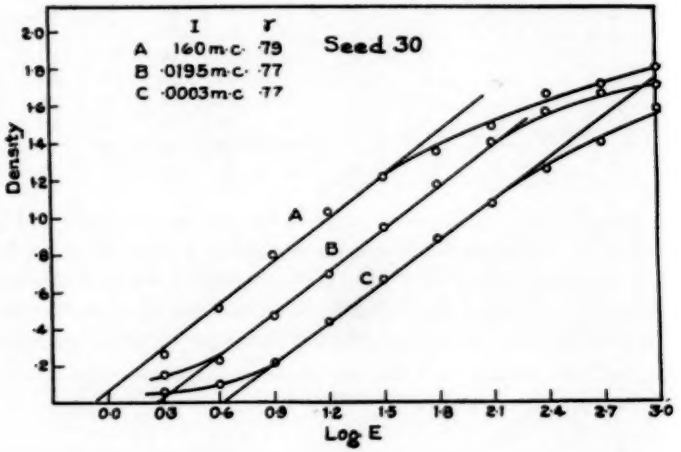


FIG. 6

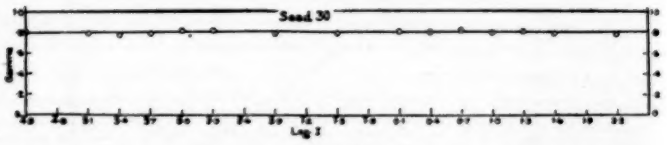


FIG. 7

will not be given here since, with very few exceptions, the conditions were identical to those contained in Table 3 for Seed Graflex plates.

Specific tests were again made to determine whether or not any γ variation, due to the intensity factor, could be measured. Fig. 6 shows characteristic curves for three widely different values of intensity. It will be noted that there is no measurable difference in the slope of these three curves. It was, therefore, considered allowable to average

the values of γ obtained from the original curves for this material, and to correct all the various curves to this γ in order to smooth out density variations due to slight experimental errors. Fig. 7 shows that there is no measurable variation in γ with intensity.

By a method exactly similar to that described for Seed Graflex, the intersection point for Seed 30 was determined. The graphical determination of this value, $D = -.07$, is shown in Fig. 8.

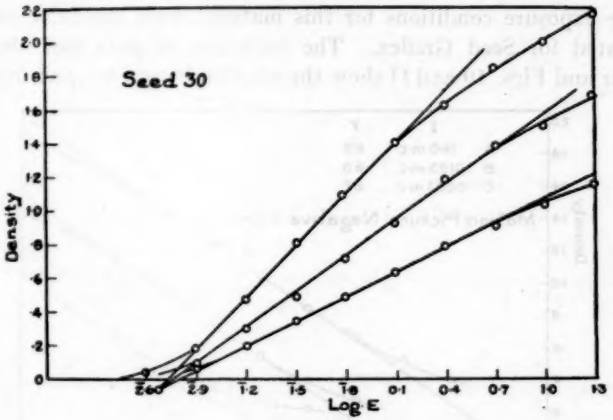


FIG. 8

The original experimental curves for this material were then corrected for γ and mean shape after the method previously described. The amount of correction was of about the same order as shown in Fig. 4 for Seed Graflex. From these corrected curves the necessary density values were obtained and the final D-log I curves shown in Fig. 9 were constructed.

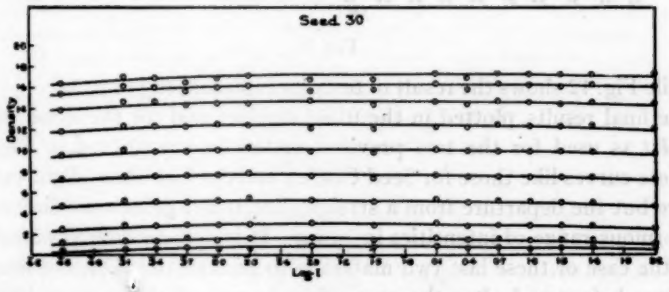


FIG. 9

The curvature in the case of this material is of a type similar to that of Seed Graflex but is somewhat less than that for the Seed 30 reported in the first paper. It is felt that this difference is due to more refined experimental work and hence that the results given here are somewhat more reliable than those published previously. The $I \cdot t$ values for these curves are identical to those for Seed Graflex.

RESULTS OBTAINED WITH MOTION PICTURE NEGATIVE FILM

The exposure conditions for this material were the same as those tabulated for Seed Graflex. The reduction of data was also very similar and Figs. 10 and 11 show the relation between γ and intensity,

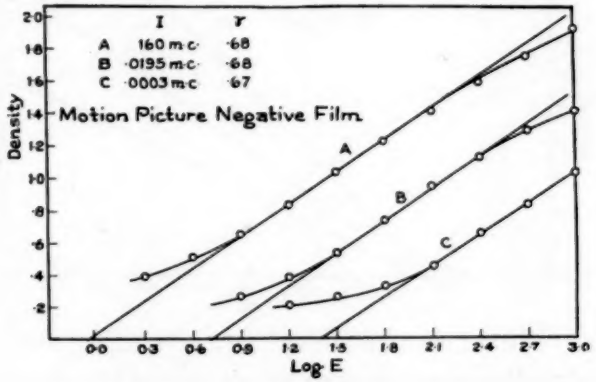


FIG. 10

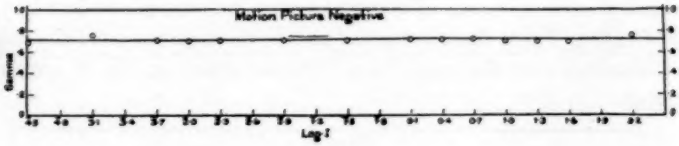


FIG. 11

while Fig. 12 shows the result of tests for intersection point, $(D = -.24)$. The final results, plotted in the usual manner and for the same values of $I \cdot t$ as used for the two previous materials, are plotted in Fig. 13. These curves like those for Seed Graflex and Seed 30 show slight curvature but the departure from a straight line is not great considering the enormous range of intensities included. It is not considered necessary in the case of these last two materials to present the tables of density values before and after the corrections for γ and shape were applied.

INTERPRETATION OF RESULTS

Before proceeding with an analysis of the data presented in the preceeding pages, it may be well to call attention to a confusing condition which exists in the terminology of this subject. In some cases the word "exposure" E is used to designate the product of intensity by time

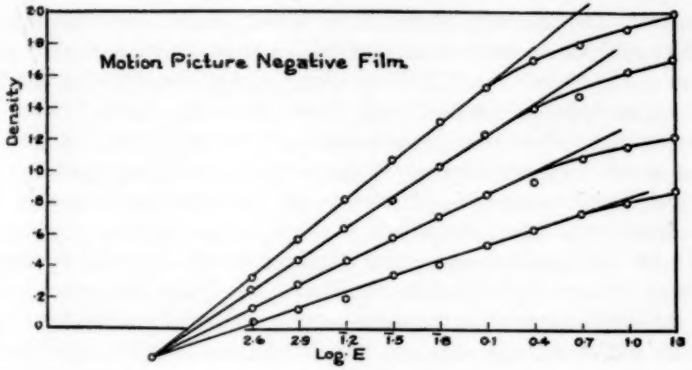


FIG. 12

which is expressible in physical units and is entirely independent of the photographic material itself. In other cases this same word "exposure" and the same symbol is used to designate the effect resulting from the

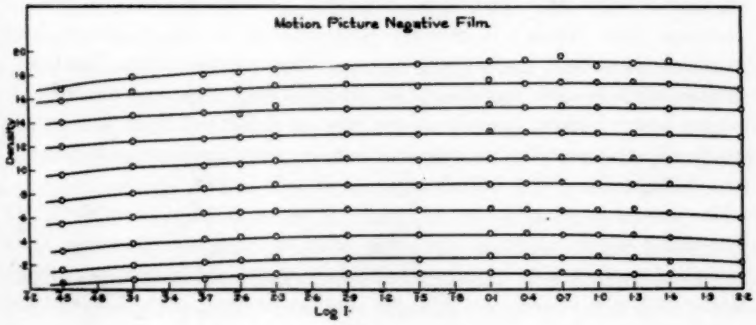


FIG. 13

action of an intensity, I, operating for the time, t. Thus the same word is sometimes used to designate both the cause and the effect. This is obviously confusing practice and for the sake of precision of terminology this double usage should be avoided.

In order to make a rational decision as to the most useful and logical terminology let us consider briefly the interrelations existing between the factors involved. Stated in its simplest terms the photographic reaction may be said to consist of a "response" following the action of a "stimulus" on a "receptor," the photo-sensitive material. The adequate stimulus is radiant energy of certain frequencies or wave lengths. The response consists of a "latent image" the magnitude of which can only be determined after this latent image has been converted into a "developed image." If we consider that the essential function of a photographic material is the production of a visible image, one having light absorbing characteristics, it seems entirely logical to regard this "developed image" as an essential part of the response of the photo-sensitive material. However, the immediate response is undoubtedly the latent image and it is convenient for many purposes to treat this latent image entirely independently from the developed image. It seems quite logical, therefore, to designate the latent image as the initial response of the photo-sensitive material and the developed image as the ultimate response. As stated previously the latent image can only be measured indirectly, that is, in terms of the resultant developed image. The magnitude of the developed image may be specified either in terms of the mass of metallic silver per unit area or in terms of the light absorbing characteristics of the silver deposits. This light absorbing characteristic is commonly expressed as the optical density D. The data available on the subject at present indicate that density is directly proportional to the mass of silver per unit area but, since there is a question whether this direct proportionality is valid for the entire range of materials and densities, it seems best to adopt density as the most satisfactory term with which to specify the magnitude of the ultimate response, and hence indirectly, the magnitude of the initial response, latent image.

As stated previously the adequate stimulus of photographic action is radiant energy and this obviously must be expressed in energy units or in units which may be converted into or expressed in terms of energy units. It is usual to express this stimulus as a product of the illumination (intensity) incident on the photo-sensitive surface by the time (exposure time, t) during which it is allowed to act. Illumination is commonly expressed in meter candles which is a quantity evaluated in terms of the sensitivity of the human eye. The product of illumination by time is, therefore, expressed in meter candle seconds (mcs). A knowledge of the visibility function of the human eye, mechanical equivalent of

light, and the spectral distribution of energy in the radiation under consideration permits the conversion of meter candles into ergs per unit area per second and hence of meter candle seconds into ergs per unit area (cm^2).

An examination of photographic literature shows that the word "exposure" has been used extensively to designate this stimulus, the product of illumination (intensity), I, by exposure time, t. This usage should be continued and some other term adopted to designate the photographic effectiveness of the exposure.

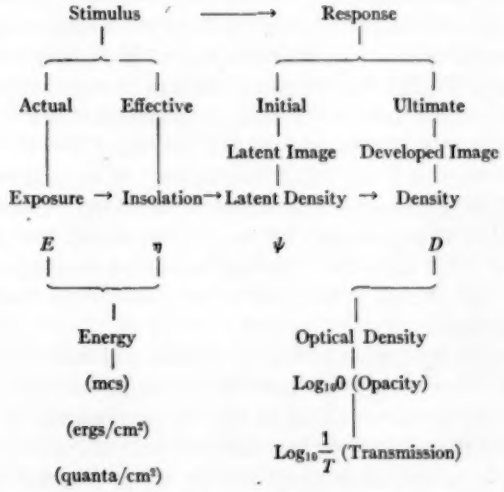
When Bunsen and Roscoe first formulated the so-called reciprocity law of photochemical action they stated that the "insolation" was equal to the product of the intensity by the exposure time. There seems to be little doubt that they intended to use this word "insolation" as indicative of the magnitude of the photochemical effect produced. This conclusion seems entirely logical when we consider that the statement "exposure is equal to intensity times time" does not constitute the expression of a law but a mere statement of a definition and, since it has always been considered that the statement made by Bunsen and Roscoe constitutes the expression of a natural law, it must be concluded that they used the word "insolation" as designating the effectiveness of the exposure. The authors, therefore, propose that this word "insolation" be adopted for use in referring to the photographic effectiveness of the exposure. It seems to us that the usage of this term by Bunsen and Roscoe constitutes ample precedence for its employment in this sense.

If the reciprocity law as stated by Bunsen and Roscoe were valid it would not be necessary to differentiate between the exposure and the insolation since the latter would be directly proportional to the former. The fact that the reciprocity law does not hold shows that the ability of the photo-sensitive material to convert energy (exposure) into available latent image depends upon the absolute value either of I or of t or of both these factors. There seems to be little doubt (at least under fixed conditions of development and quality of exposing radiation) that there is a value of intensity which for a given value of exposure produces a maximum effect. This intensity has been termed by Kron the "Optimal intensity," I_0. The efficiency with which the photo-sensitive material utilizes the incident energy is a maximum for this optimal intensity, and for all other values of intensity the efficiency, with which the plate utilizes the incident energy, is somewhat less than for the optimal intensity. If we consider that the efficiency, with which

the energy is utilized at optimal intensity, is unity, then for all other values of I the efficiency is less than unity. This value may be designated as the "coefficient of utilization" and is the value by which exposure must be multiplied in order to obtain the insolation, eta, (η). For certain purposes it is convenient to regard the failure of the reciprocity law from this standpoint and for the sake of brevity the authors propose the use of the term "utilance" instead of the phrase "coefficient of utilization." This may be designated conveniently by the Greek letter rho (ρ). We may then write

$$\text{Exposure } (E) \times \text{rho } (\rho) = \text{insolation } (\eta).$$

The value of utilance will depend, of course, upon the absolute values of I and t.



Classified under stimulus we have exposure and insolation, while under response we have the initial response which is the latent image, and the ultimate response which is the developed image. The exposure, E, is equal to the product of illumination by time and is expressible in energy units and may be specified either in terms of meter candle seconds or ergs per unit area. The insolation η has the same dimensions as exposure and is expressible in analogous units. The latent image ψ is not directly measurable and can only be expressed indirectly in terms of the developed image. The developed image can be measured and specified

directly in terms of its optical density, D. The functional relationships between these terms may be expressed as follows:

$$D=f(\psi)$$
$$\psi=\phi(\eta)$$
$$\eta=\theta(I\ ,\ t)$$
$$E=I\cdot t$$

If the procedure used in converting the latent into a developed image be specified and constant it is justifiable to assume that ψ is directly proportional to η and while it may not be possible to express quantitatively the constant of proportionality, such an assumption makes it possible to determine experimentally the relationship between exposure and insolation from values of density obtained under definite conditions of exposure and development.

In the previous communication on this subject an attempt was made to apply Schwarzschild's equation to the experimental results. This equation expressed in the above terminology is, $\eta=I\cdot t^p$, where p is a constant for any particular photographic material. It was shown definitely in the paper referred to that p was not constant and hence the Schwarzschild equation is not adequate to express the relation between insolation and exposure.

Some years ago Kron[3] published the results of an extensive research on this subject and proposed the following formula which he found to fit very closely the curves plotted from his experimental results.

$$\eta=I\cdot t\cdot 10^{-a\sqrt{\left[\log\frac{I}{I_0}\right]^2+1}} \qquad (1)$$

In this equation, a and $\log I_0$ are constants for any photo-sensitive material; a, indicating the acuteness of Kron's "Curves of Constant Blackening," and $\log I_0$ the "optimal intensity," which is defined by Kron as that intensity at which the exposure $I\cdot t$ required to produce a constant density is a minimum. Kron's results were plotted in the form of curves showing the relation between log exposure, $\log I\cdot t$, and the logarithm of the intensity, $\log I$, for a constant density ($D=$ constant), and referred to by him as curves of constant density. These curves are convex with respect to the axis of $\log I$ and hence show graphically the value of "optimal intensity" as defined by him.

[3] Kron, Eder's Jahrbuch, p. 6; 1914.
Kron, Density law of photographic plates. Publication No. 67 from the Astrophysical Observatory at Potsdam, 1913.

The authors have found it more convenient from an experimental standpoint to plot the density D resulting from constant exposure $I \cdot t$ as a function of $\log I$. The resultant curves, in most cases, are slightly concave with respect to the axis of $\log I$.

The maxima of these curves occur at that value of intensity which, for constant exposure $I \cdot t$, yields a maximum density. From this standpoint, therefore, "optimal intensity" is defined as that intensity which, for constant exposure, results in a maximum density. While this definition is different than that of Kron, it can be shown that the value of "optimal intensity" I_0 is the same for the two cases.

A comparison of the curves obtained by plotting the data in this form with those of Kron, indicates that an expression similar in form to that of Kron's should fit the experimental values. Placing $I \cdot t = E$, and writing the Kron expression in the logarithmic form, we have

$$\log \eta = \log E - a\sqrt{\left[\log \frac{I}{I_0}\right]^2 + 1} \tag{2}$$

By substituting density D for $\log \eta$ and a constant K for $\log E$, the following empirical expression was obtained:

$$D = K - a\sqrt{\left[\log \frac{I}{I_0}\right]^2 + 1} \tag{3}$$

This expression has been found to fit fairly well the curves of density for constant exposure for some of the photographic materials examined, although in cases where these curves are not of the symmetrical type, the agreement is not satisfactory.

In the curves as plotted density values are ordinates (y) and $\log I$ are abscissae (x), hence

$$K - y = a\sqrt{(x - \log I_0)^2 + 1} \tag{4}$$

This formula is applied to these data in the following way: Consider its application to curve A, of the group of D-$\log I$ curves for Seed 30. The first step in the procedure is to choose on curve A two equal density values which are as widely separated as possible, in this case—

$$\text{at } \log I = \bar{3}.9 \ (-2.1), \quad D = 1.70$$
$$\text{and } \log I = 2.0 \quad , \quad D = 1.70$$

Substituting these values in (4) we get

$$a\sqrt{(-2.1 - \log I_0)^2 + 1} = K - 1.70 \tag{5}$$

$$a\sqrt{(\ 2.0-\log I_0)^2+1}=K-1.70 \qquad (6)$$

The simultaneous solution of these two expressions eliminates "a" and "K" and when cleared of radicals allows for the solution of "log I_0."

$$\log I_0=-0.05 \text{ (log units } \bar{1}.95)$$

To get "a" and "K":

$$\text{at } x=-0.05 \qquad y=1.72$$

Again using equation (4) we have:

$$a\sqrt{(-0.05+0.05)^2+1}=K-1.72$$

from which

$$a=K-1.72$$

and

$$K=a+1.72$$

To obtain another expression between "a" and "K" put the value of log I_0 (-0.05) in equation (5) then

$$a\sqrt{(-2.1+0.05)^2+1}=K-1.70$$

$$a\sqrt{5.20}=k-1.70$$

$$2.28a=K-1.70$$

Now, substituting for "K" the value obtained above we have

$$2.28a=a+1.72-1.70$$

$$1.28a=.02$$

$$a=.0156$$

$$K=.0156+1.72=1.736$$

Returning to the formula

$$a\sqrt{(\log I/I_0)^2+1}=K-y$$

we find y for each log I value with the aid of the numerical values of the three constants, "a," "K," and log "I_0."

Table 6 (Seed 30—Curve A) contains in the column marked $K-a\sqrt{(\log I/I_0)^2+1}$, the values of D computed by the above formula, for each intensity value, and in the column headed *obs D*, the actual density values found experimentally. Column ΔD shows the density deviation of the computed values from the observed. This procedure was followed for the remaining D-log I curves of the Seed 30 set and in

TABLE 6. *Seed 30*

$\log I$	No.	$\log I - \log I_0$	$\left(\log \frac{I}{I_0}\right)^2+1$	$\sqrt{\left(\log \frac{I}{I_0}\right)^2+1}$	$a\sqrt{\left(\log \frac{I}{I_0}\right)^2+1}$	$K-a\sqrt{\left(\log \frac{I}{I_0}\right)^2+1}$	Obs. D	ΔD
4.5	−3.5	−3.45	12.90	3.59	.056	1.680	1.64	.040
3.1	−2.9	−2.85	9.12	3.02	.047	1.689	1.67	.019
3.4	−2.6	−2.55	7.50	2.74	.043	1.693	1.68	.013
3.7	−2.3	−2.25	6.06	2.46	.038	1.698	1.69	.008
2.0	−2.0	−1.95	4.80	2.19	.034	1.702	1.70	.002
2.3	−1.7	−1.65	3.72	1.93	.030	1.706	1.71	−.004
2.9	−1.1	−1.05	2.10	1.45	.023	1.714	1.72	−.006
1.5	−0.5	−0.45	1.20	1.10	.017	1.719	1.72	−.001
0.1	0.1	0.15	1.02	1.01	.0157	1.720	1.72	.000
0.4	0.4	0.45	1.20	1.10	.017	1.719	1.72	−.001
0.7	0.7	0.75	1.56	1.25	.0195	1.717	1.72	−.003
1.0	1.0	1.05	2.10	1.45	.023	1.713	1.71	.003
1.3	1.3	1.35	2.82	1.68	.026	1.710	1.71	.000
1.6	1.6	1.65	3.72	1.93	.030	1.706	1.70	.006
2.2	2.2	2.25	6.06	2.46	.038	1.698	1.70	−.002

Table 7 are tabulated the ΔD values of each intensity value for curves A to J inclusively. The mean density deviation for this entire set of data, involving ten complete curves, which cover an intensity

TABLE 7. *Seed 30*

	ΔD									
log I	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
4.5	.040	.042	.024	.036	.007	.000	.001	.005	−.008	−.012
3.1	.019	.020	.013	.012	−.007	−.007	−.013	−.010	−.022	.003
3.4	.013	.014	−.002	.006	−.011	−.010	−.015	−.013	−.024	−.009
3.7	.008	.008	.002	.009	−.011	−.013	−.012	−.016	−.022	−.009
2.0	.002	.001	−.004	.002	−.015	−.007	−.010	−.010	−.020	−.015
2.3	−.004	−.004	.004	−.005	−.010	−.001	−.009	−.013	−.013	−.011
2.9	−.006	−.007	−.003	−.001	−.003	−.002	−.003	−.005	−.003	−.003
1.5	−.001	−.002	.000	.000	.000	.000	.000	.000	−.001	−.002
0.1	.000	.0003	−.001	.002	−.004	.004	.003	.005	−.003	−.002
0.4	−.001	.0098	−.004	.005	.002	−.001	.007	.005	−.003	−.005
0.7	−.003	.008	.003	.012	−.002	.003	.000	.005	−.005	−.003
1.0	.003	.005	−.001	.009	.003	.007	.003	−.002	−.007	−.002
1.3	.000	.002	−.005	.016	−.002	.000	−.005	−.003	−.009	−.006
1.6	.006	.008	.001	.022	.002	.003	−.003	−.005	−.002	−.001
2.2	−.002	.001	.002	.006	.000	.000	.001	.000	−.018	.000
	Mean deviation .0014									

TABLE 8. *Motion Picture Negative Film*

	ΔD									
log I	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	0	*J*
4.5	.077	.038	.062	.013	.064	.061	.026	.037	.033	.031
3.1	.029	.015	.032	−.005	.026	.024	−.007	.007	.008	.012
3.7	.010	.022	.011	−.018	.003	.006	−.010	−.004	.003	.003
2.0	.000	.000	.066	−.002	−.001	.002	−.012	−.004	.000	−.007
2.3	−.001	−.002	.000	−.022	−.006	−.002	−.009	−.005	−.003	−.007
2.9	−.001	.003	−.001	−.012	−.005	−.001	−.006	−.001	.000	−.008
1.5	.004	.005	−.004	−.005	−.003	−.003	−.012	.000	.003	−.002
0.1	.005	.000	−.002	.000	.000	−.003	−.001	−.002	.000	−.001
0.4	−.001	−.001	.003	−.001	−.002	−.001	.005	−.007	−.003	.002
0.7	−.006	−.005	.009	−.005	.005	−.004	.009	−.006	−.007	−.001
1.0	−.013	−.011	.006	−.006	.001	−.008	.010	−.002	−.012	.000
1.3	−.011	−.012	.008	−.007	.004	−.003	.010	−.021	−.009	−.005
1.6	−.020	−.014	.009	−.009	.004	−.008	.013	−.020	−.005	−.005
2.2	−.010	.000	.000	−.020	.000	.000	.023	.001	.000	.000
	Mean deviation .0028									

range of approximately 1 to 500,000, is .0014, a density value smaller than one can determine precisely on any visual photometer for the measurement of photographic density.

This same procedure was followed for the other materials and the deviations between observed and computed densities for Motion Picture Negative film and Seed Graflex plates are contained in Tables 8 and 9. It will be noted also, in the case of these materials, that the agreement between the observed and computed density values is excellent. There are one or two cases in which the deviation is as high as .07 in density units, but these cases are rare and where they do exist the difference is but slightly greater than the possible experimental error.

TABLE 9. *Seed Graflex*

	ΔD									
log I	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
4.5	.002	.001	.005	.015	.000	.000	.001	.001	.001	.000
3.1	−.005	−.002	−.008	.000	−.017	−.017	−.017	−.018	−.019	−.004
3.7	−.011	−.033	−.022	−.005	−.022	−.022	−.023	−.029	−.026	−.032
2.0	−.022	−.031	−.021	−.016	−.023	−.023	−.025	−.032	−.030	−.028
2.3	−.023	−.031	−.032	−.058	−.026	−.026	−.029	−.037	−.035	−.030
2.9	−.024	−.041	−.033	−.016	−.029	−.029	−.033	−.036	−.035	−.032
1.5	−.036	−.042	−.037	−.016	−.036	−.026	−.031	−.036	−.030	−.036
0.1	−.028	−.046	−.033	−.016	−.026	−.026	−.031	−.037	−.032	−.031
0.4	−.029	−.034	−.024	−.018	−.020	−.020	−.017	−.034	−.032	−.024
0.7	−.023	−.035	−.026	−.011	−.024	−.014	−.012	−.019	−.027	−.021
1.0	−.020	−.028	−.019	−.006	−.009	−.009	−.012	−.015	−.023	−.019
1.3	−.017	−.019	−.011	−.001	−.013	−.013	−.011	−.010	−.019	−.016
1.6	−.012	−.013	−.006	.005	−.009	−.009	−.008	−.007	−.016	−.014
2.2	−.005	−.006	.000	.011	−.047	−.047	−.003	−.003	−.012	−.012
	.000	.000	.005	.015	.000	.000	.000	.000	−.010	−.010

Mean deviation −.012

TABLE 10.

Curve	*Seed 30*		*Negative Film*	
	a	log I_0	a	log I_0
A	.0156	1.95	.0376	0.10
B	.0145	0.05	.0200	0.10
C	.0160	1.60	.0167	0.25
D	.0120	1.35	.0300	0.10
E	.0207	1.45	.0211	0.00
F	.0250	1.35	.0219	0.05
G	.0296	1.35	.0307	1.60
H	.0259	1.45	.0355	1.70
I	.0284	1.35	.0265	1.90
J	.0168	1.60	.0194	1.85
Mean	.0205	1.55	.0269	1.95

Table 10 contains a tabulation of the values of "a" and log I_0 for each of the D-log I curves of two of the materials investigated.

It should be stated at this point that in the case of Seed Graflex, the log I_0 value, which represents the intensity at which the maximum photographic effect was produced, was assumed to be the maximum intensity value used in the exposure of the photographic materials. This was necessary since the D-log I curves for this material showed the maximum density at that point. That this assumption was justifiable is supported by the close agreement between the observed and computed values in Table 9. If this assumption were not valid the density deviations would be expected to be considerably greater. However, the authors do not feel justified in presenting in Table 10 the values for "a" and log I_0 on Seed Graflex since further exposures will be necessary to enable a more accurate determination of log I_0.

CONCLUSION

The results obtained thus far may be summarized as follows:

A further study of the failure of the reciprocity law has shown considerable advance toward its solution and has quite definitely established the fact that there is a definite failure of the reciprocity law. It has previously been shown by the authors that the Schwarzschild equation did not hold. An empirical expression, similar in form to that applied by Kron to his "curves of constant density," has been found to fit very well the density-log I curves of constant exposure for some of the photographic materials studied. In the case of one of the materials investigated this formula could not be applied with the same certainty as with the others, since in making the exposures, sufficiently high values of intensity to make possible the definite determination of log I_0 were not used.

The evidence seems to be very conclusive that in the case of some materials a fixed development time gives curves of which the gamma value is dependent upon the absolute value of intensity. In these cases gamma becomes less for the lower intensities but a corresponding decrease of gamma at the high intensity end of the scale has never been observed.

It is hoped that when complete data are available on a greater variety of photographic materials that it will be possible to construct an expression which will be of more general application than those thus far proposed.

Further work is planned on Seed Graflex plates and also on other photographic materials which will cover practically the entire range of photographic sensitivity, with the purpose in mind of establishing a correlation between the values of I_0 and sensitivity, since it has been previously suggested that there may be some such correlation. Work is also contemplated on color sensitive photographic materials to study the degree of failure of the reciprocity law at different portions of the spectrum.

In conclusion the authors wish to acknowledge their indebtedness to Dr. F. E. Ross for his valuable assistance and advice in connection with this problem.

COMMUNICATION NO. 233
FROM THE RESEARCH LABORATORY OF THE
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MARCH, 1925.

Method for Rapid Ray Tracing through an Optical System.—The formulas habitually used for calculating the successive coordinates of a light ray are arranged for use with logarithms, which involves the presence of a number of parameters that are not indispensable for determining the position of the ray, and that lengthen the calculation needlessly. The speed of calculation may be considerably increased by:

(1) Eliminating a certain number of unnecessary parameters, and by choosing the others in such a way that simple formulas, suitable for use with a calculating machine, result:

(2) Using special nomograms based on the formula. $u'=u+i'-i$, in which u is the angle between the ray and the optical axis before refraction, u' the angle between the ray and the optical axis after refraction, and $i'-i$ is the deviation of the ray.

This additive form makes it possible to represent a function of three variables on a plane, which is in general impossible. Moreover, the precision is sufficient for practical purposes (5 decimals). Six operations of the usual method are replaced by a single reading similar to that made on a slide rule in the multiplication of two numbers.

By combining the use of a calculating machine with that of the nomograms sixteen operations necessary in the usual method of tracing a ray through a surface, are replaced by three operations—two readings of the calculating machine and one on a divided scale.

Trials with a model have shown that the expected precision was obtained. The time required is six to ten times less than with the usual method. [H. H. Slussareff, Trans. Opt. Inst. Leningrad. *4*, No. 30, 1925. (In Russian with the above summary in French.)]

G. W. MOFFITT

CHARACTERISTICS OF PHOTOGRAPHIC MATERIALS IN THE ULTRAVIOLET

By George R. Harrison[1]

ABSTRACT

In order to develop a method of heterochromatic photographic photometry which can be used to compare line intensities at different wave lengths in the ultraviolet, the sensitivity, speed, contrast, scale, and fogging characteristics of six standard emulsions were studied in the region 4350-2144A. Light from a constant source (mercury vapor lamp or cadmium spark) was dispersed in a high aperture monochromator and the energy distribution was measured by a thermopile and d'Arsonval galvanometer. The light was diminished in intensity uniformly at all wave lengths by means of a diaphragm and screens. Density-log intensity and density-log time curves were plotted at fourteen wave lengths for each emulsion, in some cases for several times of development, and from these the values of intensity contrast and time contrast were measured. All plates showed a decreasing contrast with decreasing wave length, although some showed a minimum at 2800A, with somewhat increased contrast for a short distance at shorter wave lengths. All showed an increased speed at shorter wave lengths over that in the visible as far down as 2500A, when the speed for all except the Schumann plates decreased rapidly. Cramer contrast process plates, with extremely high contrast in the visible, showed greatly decreased contrast at shorter wave lengths, though even here the contrast was greater than that of the other plates. The reciprocity law was found to hold very closely with this plate at all wave lengths above 2500A (and possibly below), over the time and intensity range studied. Eastman Speedway plates showed fairly uniform speed and contrast down to 2500A. These two plates were found excellent for photometry, the first having high contrast and slow speed, and the second low contrast and high speed. They showed the least tendency to acquire chemical fog of any of the plates. The Schumann plates were found unsuitable for photographic photometry, having large variations in sensitivity over their surfaces. Data are also given for several oil-coated emulsions in the region 2700-2144A. Oil can be used not only to increase speed and contrast at wave lengths below 2500, but to make these factors vary less with wave length.

INTRODUCTION

Within the past few years a number of methods of photographic photometry have been developed to such a point that it is now possible to make certain types of measurements in the region 5500-2400A with very satisfactory precision. Only relative values of light intensity at a given wave length can be determined by these methods, however; they have been principally used in the determination of absorption coefficients and in the study of intensity variations in a given line.

The increased importance of intensity measurements in modern spectroscopy has made it very desirable to obtain a simple and accurate

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means of comparing the intensities of lines of differing wave lengths. To do this requires a knowledge of the sensitivity of the photographic plate throughout the spectral region used. The writer has been unable to find in the literature any determination of sensitivity and speed characteristics of photographic emulsions for wave lengths below 4000A. The small amount of sensitometric work which has been done in the ultraviolet has apparently been restricted to a few scattered determinations of contrast for one or two emulsions,[2] and to threshold phenomena.[3]

The present paper records briefly the more important results obtained in a study of the sensitivity, speed, contrast, scale, and fogging characteristics of six standard emulsions in the region 4350-2144A. The principal object of the investigation was to find whether the photographic method could be made adequate for heterochromatic photometry in the ultraviolet, and if so, to find which emulsions give the most satisfactory results for different purposes.

A variety of terms has arisen in the photographic field, many of which have different meanings as used by different writers, so a few definitions will be given for use in the present paper. An emulsion exposed to monochromatic light of intensity I for a time t, shows on development a density d, which is the common logarithm of the opacity O. O in turn is the ratio of the light incident on the developed plate to that transmitted by it. Plotting d against log I, with t constant, we get a characteristic curve; the slope of the straight portion of this curve is the intensity contrast, or γ; plotting d against log t, with I constant, we get a somewhat similar curve, whose straight-line slope is the time contrast or γ'. The so-called Schwarzschild constant, p, is the ratio of γ' to γ; it is constant in certain cases only. The d-log I curve strikes the log I axis at a point i, referred to as the intercept, whose anti-logarithm is the inertia of the emulsion for that wave length, the reciprocal of this being proportional to the speed. The threshold sensitivity is given by the point of minimum intensity τ producing appreciable blackening; i.e., the point where the actual d-log I curve strikes the log I axis. In the present paper one emulsion will be said to be more sensitive than another, or than the same emulsion at a different wave length, when the density produced by a given intensity in the first is greater than that produced in the second, all other conditions being the same. This is purposely a very loose definition; the sensitivity thus defined may be

[2] Buisson and Fabry, Rev. d. Optique, *3*, p. 1; 1924.
Harrison and Hesthal, This Journal, *8*, p. 471; 1924.
[3] Helmick, This Journal, *9*, p. 521; 1924.

expected to vary with wave length, time, intensity and development. Thus when the amount of light to which the plates are exposed is cut in half the second emulsion may be found more sensitive than the first. Finally, the scale of a plate is the intensity range subtended by the straight-line portion of the d-log I curve on the log I axis.[4]

For the simplest application of the photographic plate to heterochromatic photometry, we should like a plate of uniform sensitivity throughout the spectral region covered; that is, one in which equal intensities would produce equal densities at all wave lengths, other things being equal. This condition can in general not be attained, but it can be approached surprisingly closely in certain cases, as will appear below. Ordinarily, however, it is not sufficient to compare equal intensities, since we wish to determine relative intensities from the densities they produce. To do this completely, a determination of the characteristic surface connecting d, log I, and log t is necessary for each wave length and development process used. By keeping time and development constant, however, the problem is much simplified. Hence it will be assumed in the following discussion that all exposures can be made for the same time, except where determinations of γ' and p are to be made, and that only densities produced on the same plate are to be *directly* compared, since it is very difficult to keep the time, strength, and temperature of development constant otherwise.

EMULSIONS INVESTIGATED

The great number of excellent plates on the market makes it impossible, in a piece of work where photometry and not sensitometry is the main object, to study carefully all the types available. From an assortment of plates tried roughly, the following six were chosen for careful study as being representative:

1. Cramer Contrast Process (transparency), Green Label.
2. Seed 23.
3. Eastman Speedway.
4. Hilger Schumann.
5. Wratten and Wainwright Panchromatic.
6. Eastman Astronomical Special

In addition to these, several of the plate-oil combinations previously found most useful for photometry at short wave lengths were studied in a limited spectral region, and will be designated as follows:[5]

[4] Davis and Walters, Sci. Pap. Bur. Stds. No. 439, p. 13; 1922.
[5] Harrison, This Journal, *11*, p. 113; 1925.

1a. Plate 1 coated with medium weight machine oil (Lab. Lub.).
1b. Plate 1 coated with Nujol.
3a. Plate 3 coated with Lab. Lub.

Plates 5 and 6 were found to have similar characteristics in the more important parts of the region studied so complete data for Plate 6 will not be given. d-log I curves and d-log t curves were obtained with each emulsion for fourteen wave lengths between 4350 and 2144A, for light of known intensity at each wave length, and from these were obtained the values of γ, γ', inertia, scale, and sensitivity for each plate. All data given are representative values obtained for a number of plates coated with the same emulsion.

APPARATUS

The chief problem in work such as the present is to get monochromatic light of the desired wave lengths intense enough to be measured with a thermopile, and a means of weakening this light by known amounts to such an extent that the plates will be correctly exposed in a reasonable time. Most spectrophotometric exposures can be made in a time varying between 10 seconds and 30 minutes, and the shorter the time the better, providing uniformly timed exposures can be given. Ten seconds was chosen as the standard time in the present work for the d-log I curves; in the d-log t curves exposures of from 1 to 256 seconds were given to light of medium intensity. Since a number of plates were to be studied it was not found practicable to cover a wider range; a future paper will deal with the exact validity or failure of the various photometric laws in the ultraviolet over widely varying conditions, for the two types of plate found most useful in this work.

As a light source in the region 4350-2537A the mercury arc was used, this giving intense, widely separated lines in the region desired, and being capable of great steadiness. A preliminary study of the behavior of various types of quartz mercury vapor lamps under different conditions was made,[6,7] to determine the conditions to be chosen for greatest steadiness and ease of correction for small variations. A standard commercial arc was chosen for the present work, since very high intensites were not required. For the region 2144-2750A a cadmium spark between rotating disks was used.[8] Strictly speaking, exposures made to light from these two sources should not be directly compared, since the arc burns steadily, while the spark produces intermittent flashes, which

[6] Harrison and Forbes, This Journal, *10*, p. 1; 1925.
[7] Forbes and Harrison, This Journal, *11*, p. 99; 1925.
[8] Harrison, Phys. Rev., *24*, p. 466; 1924.

the photographic plate may not integrate in the same way that the thermopile does. When over four hundred such flashes occur per second, as in the present work, the difference seems to be negligible, however.

The spectroradiometer-sensitometer combination has been described in previous papers.[5,6] To allow introduction of the screens described below, the light source was not placed near the slit of the sensitometer, but its light was focussed on this by a 2 inch quartz lens of about four inch focal length. This was arranged so that the optical system was operating at an aperture of f 4, the lens which finally brought the spectrum to a focus being completely filled with all wave lengths concerned. It was found possible to get sufficiently large galvanometer deflections using a narrow thermopile slit and running the arc at half power to increase steadiness. With the spark the deflections were smaller, but quite reproducible.

The thermopile carriage could be removed, and in its place mounted a plate holder carrying a four by ten inch plate, arranged so that 60 spectra 9 cm long could be taken on one plate. Great care was taken to see that the thermopile moved in the same plane which the emulsion occupied when substituted for it, and the adjustment of prism and lenses was never disturbed. Exposures were timed by an electromagnetic shutter operated by a standard clock.

The experiments were done in two groups, those in which the spectrum range 4350-2537A was used, with the arc as light source, and those using the range 2750-2144A, with the spark. In the first group, with the thermopile mounted in the focal plane, the arc was started and allowed to warm up until it had come to equilibrium with its surroundings. This generally took from one-half to one hour. The arc was fed by a 150 volt D.C. generator run by an induction motor, and in series with it were placed a variable resistance, a large reactance, and an accurate ammeter, while a voltmeter was kept connected across its terminals. Conditions were always arranged so that the arc came to equilibrium drawing 3.35 amperes with 60 volts drop across the luminous column. Final adjustment to this value was made by varying the ventilation of the box surrounding the arc.

When beginning an experiment the thermopile was run through the spectrum three times, the energy distribution curve being plotted from the average readings obtained. Then the plate holder was substituted for the thermopile, the necessary screens and diaphragms were inserted, and a number of pictures were taken. Finally the thermopile was replaced and the energy measured again. The arc was being run at such

low power that no change in the transmission of the quartz was noticeable over several months of intermittent operation. The method of using the spark as a constant source has been described previously.[9] The intensities of the maxima used are given in Table 1. Absolute energy values were obtained by calibrating the thermopile with a standard lamp.

TABLE 1. *Intensities of arc and spark lines used*
(Maxima 7-15 from the mercury arc; maxima 1-6 from the cadmium spark)

Maximum	Wave-lengths Angstroms	Intensity—I (ergs/cm²/sec)	log I
7	4359–48	3200	3.50
8	4078–47	2080	3.32
9	3663–50	4420	3.64
11	3132–26	2230	3.35
12	3026–22	950	2.98
13	–2800–	300	2.48
14	–2700–	490	2.69
15	2537	570	2.75
1	2748	460	2.66
2	2573	320	2.50
3	2312–29	830	2.92
4	2265	610	2.78
5	2194	400	2.60
6	2144	380	2.58

It was found that the maximum intensity to which it was desired to expose any plate at the longer wave lengths was about 1/2000 of that required for accurate measurement by the thermopile and d'Arsonval galvanometer, while the minimum required a further reduction to 1/1800 of this value. Accordingly, it was necessary to cut down the light by means of a diaphragm and screens. Since the aperture of the spectrograph was so great corrections for zonal aberration had to be applied as outlined below. Care was taken to see that all wave lengths were reduced by the same amount when the diaphragm was used. This was placed next the prism, the illumination being very uniform. A new set of wire screens was made, the gauzes being merely blackened by heating, without application of any corrosive solution. They were carefully measured on the thermoelectric densitometer[9] while in motion, and were used in the combinations given in Table 2. In this table are also given the calibrations of the set of screens previously used,[8,10] two separate screen combinations being thus available.

[9] Harrison, This Journal, *10*, p. 157; 1925.
[10] Harrison, Phys. Rev., *25*, p. 768; 1925.

It seems to be taken generally for granted that screens of blackened wire netting are truly neutral in the visible and ultraviolet, but there still is a possibility of doubt as to the correctness of assuming that the density of a number of screens for all wave lengths is the sum of their separate densities. In an extreme case in the present work, eleven

TABLE 2. *Wire Screen Transmissions*

Screens	Transmission	log I transmitted
None	1.00	4.00
B4	.33	3.52
A1	.51	3.71
B6	.17	3.23
A5	.29	3.46
B3-B4	.10	3.00
A1-A4	.17	3.23
B3-B4-B5	.033	2.52
A1-A2-A4	.083	2.92
B3-B4-B6	.017	2.23
A1-A4-A5	.051	2.71
B2-B3-B4-B5	.0102	2.00
A3-A4-A5	.033	2.52
B1-B2-B3-B4-B5	.0031	1.49
A1-A3-A4-A5	.017	2.23
B2-B3-B4-B5-B6	.0017	1.23
A2-A3-A4-A5	.0157	2.19
B1-B2-B3-B4-B5-B6	.00053	0.72
A1-A2-A3-A4-A5	.0081	1.91

screens were used together, cutting the light down to less than 1/200,000 of its original value, and no appreciable error could be found by photographic methods so long as the screens were kept in irregular motion relative to one another. They were placed in the widest sections of the light beam, outside of the slit of the sensitometer, and two of the same number of wires per inch were never placed next each other. Each screen was hung as a short pendulum from an irregularly bent axle, so that the resulting motion was a complex one.

All plates were developed in a metol-hydrochinon solution of good keeping quality.

PLATE CHARACTERISTICS

In order to simplify computation, all plates were first exposed to the same set of intensities, the range being great enough to more than cover the straight-line portions of all characteristic curves. Ten-second exposures were made to the mercury arc spectrum cut down in ten successive steps; then on the same plate exposures were made to a medium in-

tensity for 1, 2, ... , 256 seconds. Next, both sets of exposures were repeated on the same plate. In this way two sets of data could be obtained for a given emulsion from a single plate. Another emulsion was then exposed in the same way, care being taken to see that the arc conditions, and developer strength and temperature did not change in the meantime. After a number of specimens of each type of plate had been exposed and developed, the spark was substituted for the arc and the procedure was repeated for the short-wave region.

From the densities measured on such plates curves similar to those in Fig. 1 were plotted for the fourteen intensity maxima tabulated in

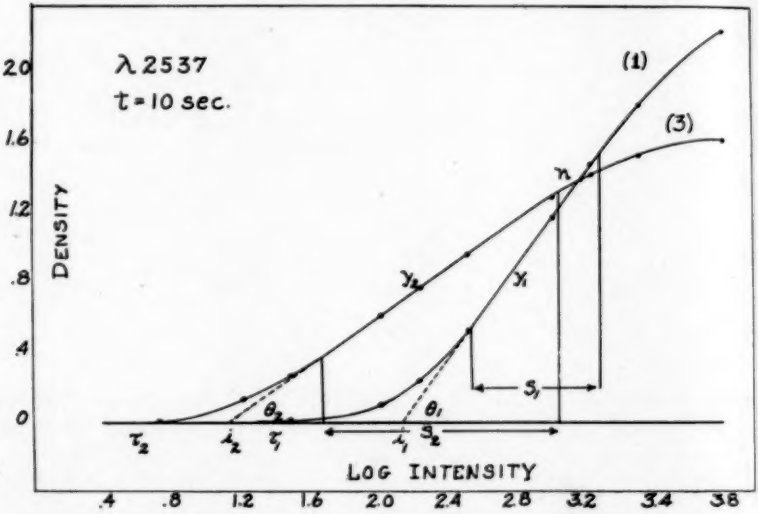

FIG. 1. *Characteristic curves for the fastest and slowest emulsions, exposed for ten seconds to light of wave length 2537A. Curve (1), Cramer Contrast Process Plate; Curve (3), Eastman Speedway Plate. The plates were developed together.*

Table 1. Although a fairly wide slit was used on the sensitometer, to give a broad rectangle for measurement at each line, the light was quite pure except at wave lengths marked —2800— and —2700— in the table. Here, however, no very abrupt changes with wave length were taking place in most of the plate characteristics, and the presence of a number of lines of intensity almost as great as the main ones made little difference. Fig. 1 shows characteristic curves for Plates (1) and (3), the slowest and the fastest emulsions tested, to light of wave length 2537A, plotted to the same absolute scale. From the values of i_1 and i_2

it is evident that at this wave length, Plate (3) is about 10 times as fast as Plate (1). The latter has much greater contrast, however. Plate (3) has greater threshold sensitivity, and also greater actual sensitivity below density 1.4, but above this density Plate (1) has the greater sensitivity. The scale of Plate (3) is much the greater of the two, not so much because its straight-line portion is longer, but because its smaller contrast causes this portion to subtend a greater distance on

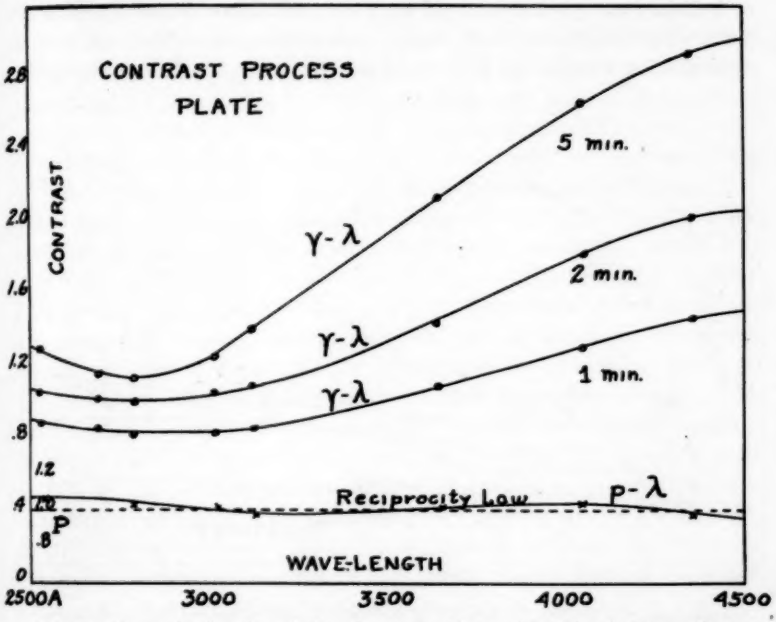

FIG. 2. *Curves showing the variation of contrast with wave length for the Cramer Contrast Process plate, for three times of development. Also curve showing the degree of departure of this emulsion from fulfillment of the reciprocity law, indicated by the dotted line.*

the log I axis. So while a five-fold range of intensities is all that one can use and stay on the straight-line portion of Plate (1), on Plate (3) a fourteen-fold range is available.

In addition to curves such as those given in Fig. 1, d-log t curves were plotted for each wave length. Values of γ' were measured from these, and used with the values of γ from the other curves to calculate p. The smallness of errors in individual screen combinations was indicated by the regularity of the d-log I curves plotted, the minor variations noted being entirely explainable by local variations in the emulsion.

In addition it should be remembered that in certain cases, where the points of highest intensity were still on the straight-line portions of the curve, the slope of this line was determined by the first two points, these being given by an exposure with no screen and with one screen respectively. Accordingly, that further additions of screens and substitutions of different screens did not change this slope appreciably indicates absence of cumulative errors.

In Fig. 2 are given curves showing the variation of intensity contrast with wave length for three times of development of Plate (1) in the region 2500-4350A. As was expected, the contrast decreases rapidly

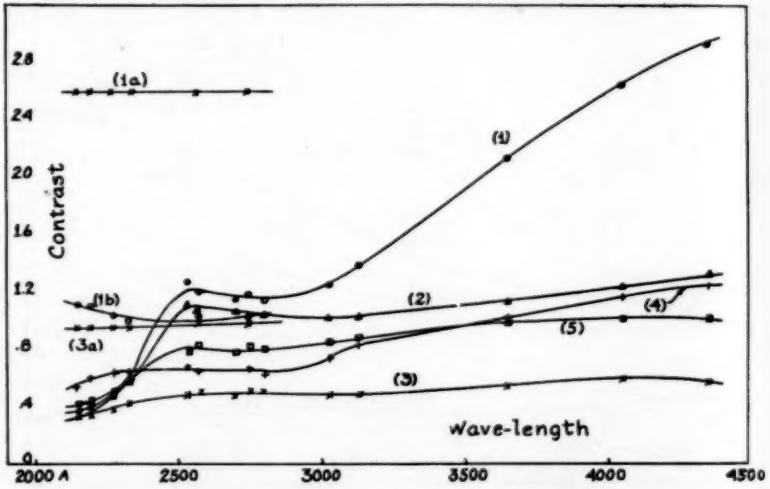

FIG. 3. *Curves showing the variation of contrast with wave length for most of the emulsions studied, all developed for five minutes except (4). (1) Cramer Contrast Process plate; (2) Seed 23 plate; (3) Eastman Speedway plate; (4) Hilger Schumann plate; (5) Wratten and Wainwright Panchromatic plate; (1a) Plate (1) bathed with Lab. Lub. oil; (1b) Plate (1) bathed with Nujol; (3a) Plate (3) bathed with Lab. Lub. oil.*

as we pass from the visible into the ultraviolet, but it reaches a minimum about 2800A, and then rises somewhat at lower wave lengths. There is a maximum near 2500, as the contrast soon falls greatly, reaching very low values at 2100A (see Fig. 3). It is evident that a decrease in development decreases the contrast to a great extent in the visible, but does not affect it so much in the ultraviolet, so that incomplete development tends to produce greater uniformity of contrast with this plate. Below these curves is given another showing the variation of p with wave

length; this was averaged from a large number of determinations with Plate 1. The dotted line shows the value of p necessary to give validity of the reciprocity law at all wave lengths; that is, if the curve coincided with this line, $\gamma' = \gamma$, and intensity and time would be equivalent in any exposure. It cannot be stated definitely whether the small variations of p from the reciprocity law are real or not. In any case the plate shows a very fair agreement with this law for exposures of 1-500 seconds, at all wave lengths studied.

In Fig. 3 are given curves showing the variation of intensity contrast with wave length for five of the plate-types studied, all having been developed for five minutes except the Schumann plate. These curves were drawn from data obtained both from plates exposed to the arc and those exposed to the spark, the two sets overlapping in the range 2537-2750A. Plate (1), the slowest, shows the greatest contrast above λ2400, while Plate (3), the fastest, shows the least contrast at all wave lengths. The latter, however, shows fairly uniform contrast throughout the spectral region. The Schumann plate, (4), is of especial interest, since it is perhaps the most fundamental type, containing little gelatine in the emulsion. Both laboratory-made and commercial Schumann plates were found unsuitable for photographic photometry, however, the sensitivity varying greatly over the surface of a given plate. This type of plate cannot be developed to high contrast on account of fogging. Contrast data on several oiled plates are included in the short wave length region. The advantage of these for certain purposes is obvious.

In Fig. 4 are plotted curves showing the variation of speed with wave length for the five most important emulsions, and also for the oiled plates at short wave lengths. When curves similar to these were first plotted, from photographs in which a diaphragm was used to cut down the light to approximately 1/2100 of its original value, with screens added for lower intensities, the writer was much surprised to find that the speed apparently increased as one passed from the visible into the ultraviolet. The relative values were as expected; thus, plate (3) was found to be more than ten times as fast as Plate (1), but all plates showed an increase of speed at shorter wave lengths. This could be accounted for by supposing the lenses to have been filled by some wave lengths when the diaphragm was removed to measure the energy, and not by others. Accordingly, the diaphragm was left out when exposing the next set of plates, and moving screens were used to cut down the light by the whole amount. The same results were obtained as in the first case.

When the aperture of the optical system was reduced from 50 mm to slightly over 1 mm, changes in energy distribution resulted at the focal plane. Accordingly, photographs were taken without the diaphragm, using a large number of screens as mentioned above. In this case the same zonal aberration was present in the photographic portion of the experiment as in the radiometric portion. The slit of the densitometer was then made the same width as the slit of the thermopile. This completely eliminated errors due to aberration and different line shapes in different portions of the spectrum. From results obtained from plates taken in this way, the required corrections, which were small, could be

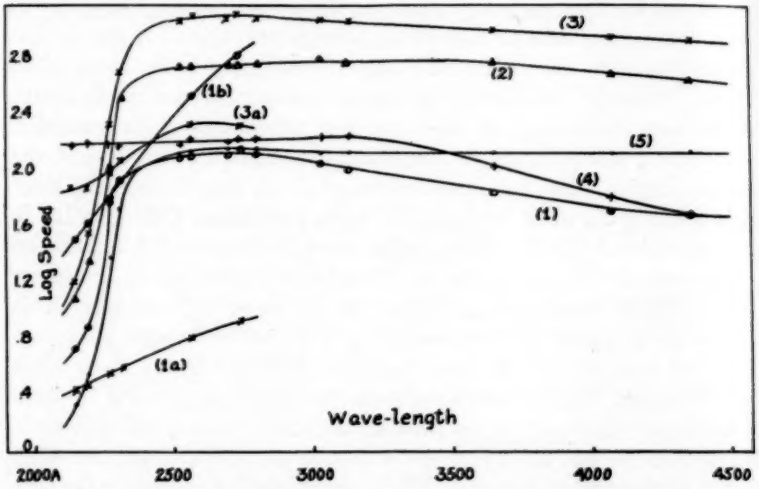

FIG. 4. *Curves showing the variation of speed with wave length for the eight emulsions shown in Fig. 3.*

found and applied to those results taken with the diaphragm in position, and all errors due to changing aperture corrected for.

Ordinarily, other things being equal, a decrease of contrast seems to mean an increase of speed except when due to under-development, since the straight-line portion of the characteristic curve swings around a point somewhere above the log I axis and throws the intercept further towards the origin. Thus, it is possible for a plate with both less contrast and less sensitivity at practically all densities than another, to have greater speed.

It is seen in Fig. 4 that the speed of emulsion (3) changes fairly uniformly above 2500A. This plate appears to be a very useful one for

certain types of photographic photometry in this region, since its characteristics are so uniform; its drawbacks are a rather low contrast, and a tendency to be fogged readily by stray light, as is to be expected from its high sensitivity to weak light. Its sensitivity is greater than that of Plate (1) at low intensities, but less at high intensities, for most wave lengths.

Plate (1) shows a great increase in speed in the ultraviolet, where its contrast falls off. It has similar speed characteristics to the Schumann plate above 2500A. Plates (1) and (3) were found to be the best suited for the kind of photographic photometry in mind, the first showing the least tendency to fog of any plate examined, the finest grain, the greatest contrast, and the greatest sensitivity at high densities. Apparently the reciprocity law is closely fulfilled by it also, which is very useful in certain cases but not at all necessary for accurate photometry. Its chief drawback is non-uniformity of contrast throughout the spectral region.

All the other emulsions showed various characteristics which made them slightly less desirable for the type of photometry desired than either (1) or (3). The dyed plates, (5) and (6), showed some unusual irregularities probably due to the added complication of the dye, and an increased tendency to fog. The Seed 23 plates seem to possess many of the disadvantages of both fast and slow plates without all their advantages.

In photometric work the tendency of a plate to acquire chemical fog is of great importance, since not only must a correction be made for the changed density of the background, involving a reduction of densitometer sensitivity generally, but fogging may tend to change sensitivity in a way difficult to correct for. A small variation of speed with time of development was found for some plates, and it is thought that this could be accounted for largely by sensitivity changes introduced through fogging, although there may be another effect also.

TABLE 3. *Chemical fog on plates*

Plate type	Transmission factor of glass plus emulsion for development of		
	1 minute	2 minutes	5 minutes
1	.84	.81	.77
2	.54	.40	.26
3	.59	.53	.45
4	.35	—	—
5	.63	.54	.41
6	.40	.20	.16

In Table 3 are given the transmission factors of the unexposed portions of various plates for different times of development, to show the degree of chemical fog found in each. Of course, these variations of fog were corrected for directly in all density determinations. It is seen that Plates (1) and (3) show the least tendency to acquire chemical fog of all the plates studied.

In Fig. 5 are given three sensitivity curves for Plate 1. These show the densities produced, with five minutes development, by energy in-

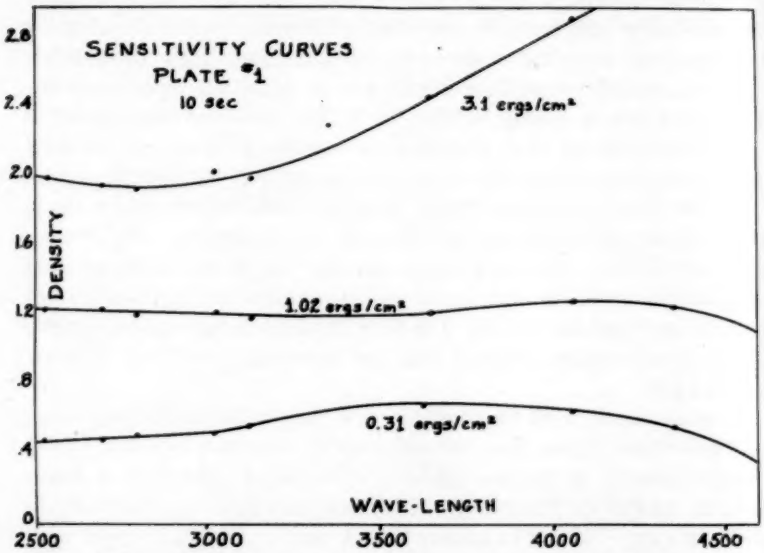

FIG. 5. *Sensitivity curves for a typical Cramer Contrast Process plate, showing the densities produced at different wave lengths by three energy densities.*

puts of 3.1, 1.02, and 0.31 ergs per square cm, for different wave lengths above 2500A. Below this point the sensitivity decreases rapidly in all three cases. It is seen that the sensitivity decreases greatly with decreasing wave length at high densities throughout the spectrum, but that at moderate densities it remains almost constant, decreasing again at low densities. Ordinarily, it might be expected that at low densities the sensitivity would increase at shorter wave lengths, from the speed and contrast values given, but this is not true because at these low intensities the d-log I curves are varying from their straight-line portions, and the longer wave lengths begin to depart from these at higher

densities than do the shorter ones. Most emulsions seem to have curves similar to the top one of Fig. 5 at lower densities; that is, in general, the sensitivity of an emulsion decreases with wave length even though the speed increases, because the decreasing contrast more than makes up for the increased speed.

From the curves of Fig. 5 we can say that roughly 0.6 ergs/sq. cm would be required to produce unit density at λ4100A in this plate. Leimbach[11] gives 0.63 ergs/sq. cm for the same value, and although the closeness of the agreement is purely accidental, since he used a different

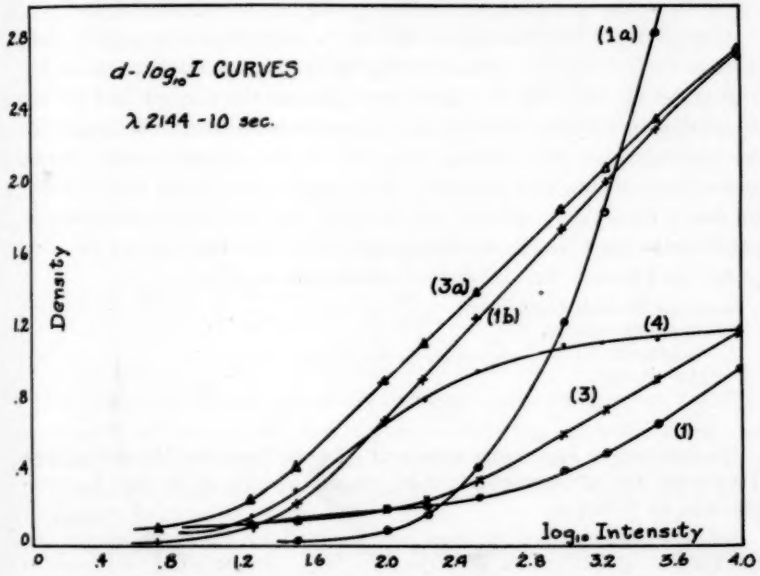

Fig. 6. Typical characteristic curves for several emulsions exposed to light of wave length 2144A for ten seconds. The emulsions are marked to correspond with those in Fig. 3.

plate and development, and a different rate of energy input, the fact that the results are of the same order of magnitude is gratifying. Jones and Schoen[12] give a value of 0.2 ergs/sq. cm for a still different plate; such determinations may be expected to vary by a factor of 10 in certain cases, however, due to different degrees of development, etc.

In Fig. 6 are given *d*-log *I* curves for six emulsions exposed to the shortest wave length studied, λ2144A. This region is of especial interest

[11] Leimbach, Zeits. wiss. Phot., 7, pp. 157, 181; 1909.
[12] Jones and Schoen, This Journal, 7, p. 213; 1923.

on account of the rapidly decreasing sensitivity of the ordinary photographic plate as the short wave length region is approached. The curves are numbered to correspond with those of Figs. 3 and 4. (1) and (3) represent the extreme cases of the average plate. The Schumann plate shows a very peculiar type of curve in (4); the straight-line portion is extremely short, and the "shoulder" is quite long. The curves for emulsions coated with oil, (1a), (1b), and (3a), illustrate the advantage of using this method of increasing sensitivity; for fairly high intensities all of these plate-oil combinations give greater sensitivity and contrast than the Schumann plate (at least at wave lengths above 2100A).

Data on scale determinations will not be included in this paper, since a more careful study of this is being made throughout the ultraviolet region for Plates 1 and 3. Since these plates, the slowest and fastest studied, respectively, were found to have such excellent characteristics for photographic photometry, they will be investigated over a much wider range of time and intensity. It is hoped that by this means it will be found possible to develop an accurate method of heterochromatic photometry such that intensities at different wave lengths can be compared throughout the visible and ultraviolet regions.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY,
CAMBRIDGE, MASS.
MAY 20, 1925.

Quantitative Determination of Aberrations by Means of the Twyman Interferometer.—The results of this work can be summarized as follows:

(1) The interference method put forward by F. Twyman for the qualitative investigation of objectives has been worked out quantitatively, and tested by experiment.

(2) As a comparison of the interference method with the Hartmann method the longitudinal aberrations and the astigmatic difference for several optical systems were determined by both methods. [K. I. Butkoff, Trans. Opt. Inst. Leningrad. *4*, No. 29, 1925. (In Russian with the above summary in German.)]

G. W. MOFFITT

OPTICAL SOCIETY OF AMERICA PROGRESS COMMITTEE ON RADIOMETRY AND PHOTOMETRY*

REPORT PRESENTED OCTOBER 24, 1924

In general this report has been planned to give a brief résumé of the progress in radiometry and photometry represented by work published during the years 1923 and 1924. It is not intended to be a bibliographical record nor to cover all papers on these subjects.

Six subdivisions are included in the report, and for each of these, one member of the committee has assumed primary responsibility. These are as follows:

(1) Spectroradiometry—W. W. Coblentz
(2) Spectrophotometry—K. S. Gibson
(3) Photometry and Illumination—E. C. Crittenden
(4) Reflectometry—A. H. Taylor
(5) Colorimetry—I. G. Priest
(6) Pyrometry—C. O. Fairchild.

I. SPECTRORADIOMETRY

Recent investigations of absorption and reflection spectra, particularly in the infra red, have been directed mainly to a search for harmonic frequency relations among the absorption (and reflection) bands, and to a detailed analysis of the fine line structure which is found in some of these bands.

Investigation of the fine structure of the absorption bands of certain gases and their applications to recent theories were summarized last year by Randall.[1]

By using a very large dispersion recent investigators have been able to resolve some of these absorption bands into numerous fine lines (40 or more). It is found that some of the bands, particularly of hydrogen chloride and of water, which happen to be closely harmonic (e.g. HCl at 1.75μ and 3.5μ; water 1.5, 3.0, and 6.0μ) are merely the envelope curves passing through the maxima of a series of fine absorption lines.

* The members of this committee in addition to those mentioned in the text are Loyd A. Jones and Carl W. Keuffel.

[1] Randall. J.O.S.A. & R.S.I., 7, p. 45; 1923.

The most recent investigations [2,3] of the HCl band at 3.5μ have dealt with effects of high temperatures on the fine absorption lines.

Recently, interest has been revived in the harmonic relations observed[4] about two decades ago, among the absorption bands of various substances; particularly hydrocarbon compounds, which were found to have maxima at the closely harmonic positions 1.71, 3.43, 6.86μ, and sometimes at about 14μ. These substances are complex in chemical constitution, and, in addition to the absorption band at 6.86μ often have close-by absorption bands at 6.3 to 6.5μ and at 7.3μ which have the effect of shifting the apparent location of the maximum which falls in the harmonic series.

A recent investigation of the upper harmonics in infrared absorption spectra[5] shows that in the case of $SrSO_4$ there is an approximately harmonic series of bands at 9.05μ, 4.5μ, 3.02μ, and 2.25μ.

Ellis[6] has investigated the harmonic frequency in the infra red absorption sprectra of liquids and solids. He observed absorption bands at approximately 1.7μ, 1.38μ, 1.17μ, 1.02μ and 0.9μ in all the carbon-hydrogen compounds examined. These bands appeared to be respectively the third to the seventh harmonic of a fundamental wave length which occurs at about 6.9μ. This relationship, however, is not truly harmonic throughout the series, and a formula is proposed which accounts for the progressive deviation toward a longer wave length in the higher frequencies of the series.

Kratzer[7] suggested a similar formula which is applicable to the approximately harmonic relationships in the absorption spectra of the diatomic gases, HCl, HBr, and HF.

The most recent contribution to this subject is by Barker[8] who finds that the bands in the spectra of hydrogen cyanide do not appear to fit into the particular harmonic relation suggested by Kratzer. The full significance of these approximately harmonic absorption bands does not seem apparent at the present time, but that such a condition obtains can no longer be doubted.

[2] Colby, Meyer and Bronk. Astrophys. Jour., *57*, p. 7; 1923.
[3] Spence and Holley, J.O.S.A. and R.S.I., *7*, p. 169; 1923.
[4] Coblentz. Investigation of Infra Red Spectra. Publication Nos. 35, 65, and 97, Carnegie Institute of Washington.
[5] Schaefer and Thomas. ZS. f. Physik., *12*, p. 330, 1923.
[6] Ellis. J.O.S.A. and R.S.I., *8*, p. 1; 1924.
Phys. Rev. (2) *23*, p. 48; 1924.
[7] Kratzer. Zeitsch. f. Physik, *3*, p. 289; 1920.
[8] Barker. Phys. Rev. (2) *23*, p. 200; 1924.

2. SPECTROPHOTOMETRY

The 1922-23 O.S.A. Committee on spectrophotometry has prepared a report[9] which, it was hoped, would serve as a basis for future progress reports. It covers the subject briefly from the beginnings of spectrophotometry to the end of 1923, including sections on Nomenclature, Spectrometry, Photometry, Visual Instruments, and Auxiliary Methods. The present report overlaps it for the year 1923. Its scope has been taken to include the development and novel use of apparatus for the visual spectral measurement of the emissive, transmissive, or reflective properties of light sources or materials, together with such auxiliary methods as may be used to supplement or extend these measurements, and miscellaneous papers on spectrophotometry which may be valuable for study or reference.

The cutting off of the ready supply of German instruments and the growing demand from commercial and scientific laboratories for a means of measuring and specifying color both added impetus to the development of spectrophotometric apparatus in this country. Two new *ensembles* appearing about the beginning of 1923 were the Keuffel and Esser color analyser and the Bausch and Lomb combined spectrometer and Martens photometer.

K. and E. Color Analyser.[10]—The novel features of this apparatus are the illumination and the variable rotating sector. The illuminating sphere, supplied for the first time on commercial instruments, enables measurements of spectral reflection under conditions of (approximately) diffuse illumination to be made as readily as measurements of spectral transmission. The variable sector enables the brightness of one beam to be continuously varied relative to the other and in such a way that transmissions and reflections are read directly on the scale of the instrument. The spectrometer is of the constant-deviation, direct-reading type.

B. and L. Spectrophotometric Equipment.[11]—The Martens polarization photometer, now being made by Bausch and Lomb, has been combined with their constant-deviation, direct-reading spectrometer to form a spectrophotometer designed primarily for spectral transmissive measurements. For this purpose the apparatus includes a light source and

[9] J.O.S.A. & R.S.I., *10*, p. 169; Feb. 1925.

[10] Keuffel and Esser Co., Hoboken, N. J., J. O. S. A. & R. S. I., *7*, pp. 85; 1923 (See Advertisements.)

[11] Bausch and Lomb Optical Co., Rochester, N. Y., J.O.S.A. & R.S.I., *7*, p. 88: 1923 (See Advertisements.)

tubes with support for the containing of solutions in addition to the parts mentioned above.

In addition to these recent developments the following instruments are available:

Nutting polarization photometer with spectrometer. These parts together with light source and tubes with support for spectral transmissive measurements, are made by Gaertner[12] in this country and Hilger[13] in England. In addition, Gaertner makes the *Brace* and *Brace-Lemon spectrophotometers*, and Hilger the *Hufner spectrophotometer*. The firm of Franz Schmidt and Haensch, Berlin, Germany also supply instruments including the *Brace*, the *Lummer-Brodhun* and the *König-Martens spectrophotometers*.

A study has been made by Harrison[14] of the characteristic density-log time and density-log intensity curves on photographic plates in the ultraviolet, along with a method for obtaining a source of constant intensity for this region. The data should be of interest for those cases where a null method can not be used.

A detailed discussion of the use of the so-called neutral wedge in photographic spectroradiometry is given by Plaskett[15] in connection with his investigation of solar and stellar energy distribution curves.

By a combination of photoelectric cell, galvanometer-equal-deflections method, and Brodhun variable sector, Gibson[16] has effected a rapid, direct-reading method of measuring spectral transmission. The same method is applicable also with the thermopile.

Improved construction in the Nichols radiometer by Tear[17] has resulted in a several-fold increase in sensitivity. The deflections are proportional to the incident radiant intensity and the instrument is being used for work in the visible and infrared.

A report has been made by Priest and others[18] on the precision and accuracy of measurements made on the König-Martens spectrophotometer as used at the Bureau of Standards.

[12] The Gaertner Scientific Corp., 1201 Wrightwood Ave., Chicago, Ill. (See Advertisements.)

[13] Adam Hilger, Ltd., 75a Camden Road, London. (See Advertisements.)

[14] Harrison and Hesthal. J.O.S.A. & R.S.I., *8*, p. 471; 1924.
Phys. Rev., *23*, p. 770, 1924.

[15] Plaskett, H. H. Dom. Astrop. Obs. Pub., *2*, p. 213; 1923.

[16] Gibson, K.S. J.O.S.A. and R.S.I., *7*, p. 693, 1923.

[17] Tear, J. D., Phys. Rev., *23*, p. 641; May 1924.

[18] Priest, McNicholas, and Frehafer. J.O.S.A. & R.S.I., *8*, p. 201; 1924.

Benford[19] has published a paper discussing several points of interest in the presentation of spectrophotometric data.

Variable rotating sectored disks have been devised by Ives[20] and Karrer,[21] in addition to the Keuffel and Esser sector mentioned above, and those devised by still other investigators in previous years. They are mentioned here as they have valuable application in spectrophotometry.

In addition to the 1922-23 O.S.A. report on spectrophotometry other papers of a more or less general nature have appeared by Twyman,[22] Wales,[23] and Gibson[24]; and Schaum[25] has published the first of a series of papers on photometry and spectrophotometry, this one dealing with photographic methods and a study of the transmission of parallel plates with diffusing surfaces.

3. PHOTOMETRY AND ILLUMINATION

To provide a satisfactory permanent basis for photometry two things are needed; first, a really reproducible standard of light, and second, agreement on some method or criterion for the comparison of the intensity of lights of different color. Some definite progress has been made in recent years toward meeting both of these needs.

With regard to a primary standard, opinion is well crystallized on the use of a "black body," or complete radiator, under some specified conditions. What these conditions shall be remains to be decided, and the decision must depend somewhat upon the certainty and precision with which the specified conditions can be reproduced.

Ives' paper on "A Primary Standard of Light Following the Proposal of Waidner and Burgess"[26] reports a complete and very precise determination of the brightness of the interior of a hollow cylinder of pure platinum at its melting point. The paper also includes a study of the optical properties of such a cylinder indicating that it can be considered

[19] Benford, F. A., Trans. I.E.S., *18*, p. 67, 1923.

[20] Ives, H. E., J.O.S.A. & R.S.I., *7*, p. 683; 1923.

[21] Karrer, E., J.O.S.A. & R.S.I., *8*, p. 541; 1924.

[22] Twyman, F., Optical Methods in Control and Research Laboratories, Sec. II, Absorption Spectra and Spectrophotometry. Adam Hilger, Ltd., *1*, 2d ed.; 1923.

[23] Wales, H., The Spectroscope and Spectrophotometer and Their Application to the Identification and Analysis of Dyes. American Dyestuff Reporter, *12*, pp. 751-755, 791-793, 855-857, 863-866; 1923.

[24] Gibson, K. S., Spectrophotometry. The Dictionary of Applied Physics, Vol. IV, R. Glazebrook, Editor, The MacMillan Co., New York, 1923.

[25] Schaum, K., Zeits. wiss. Phot., *22*, p. 148; 1923.

[26] Jour. Frank. Inst., *197*, pp. 147 and 359; Feb. and March 1924.

as a black body. Furthermore, the brightness found agrees with the black-body brightness curve of Hyde, Forsythe and Cady[27] far more closely than might be expected in view of the uncertainty of the temperature scale on which such a curve must depend. To the inherent advantages of choosing a fixed point like that of platinum independent of temperature scales, Ives' skillful experimental work has now added the weight of a direct determination of the brightness value, 55.4 candles per square centimeter.

Measurements made by Hoffman on a black body furnace at the Reichsanstalt are reported to give a brightness considerably higher than Ives found, but the reports so far published[28] also indicate an unexpectedly high value for the platinum melting point (1771°C). Possibly some explanation may be found for both high values.

The most important recent paper in the field of photometry of different colors is Gibson and Tyndall's[29] on "The Visibility of Radiation." This gives data found by the step-by-step method, but in combination with earlier work using other methods the results are generally accepted as establishing a reliable curve which is really characteristic of the central portion of the retina of the average eye. So long as one uses a small field (about 2°) with brightness not far removed from the proposed standard value (2.5 millilamberts), measurements by either flicker or equality-of-brightness ought to agree fairly well with results calculated from this curve. The average data recommended have already been accepted by the International Commission on Illumination.[30]

Such acceptance of a standard visibility curve, instead of a specified procedure for heterochromatic measurements, increases the importance of Priest's method of equalizing colors by means of adjustable quartz-Nicol combinations for which the total transmission can be calculated from a visibility curve. The recent publication[31] of a complete description of this instrument is therefore very timely.

Another instrument in which visibility data are actually used to avoid the difficulties of heterochromatic photometry is Ives' "Color-Match Photometer for Illuminants."[32] In this case, spectral transmission curves

[27] Phys. Rev. (2) *13*, p. 45; 1919.

[28] ZS. f. Phys. *27*, p. 285; 1924.

[29] Bur. Stds. Sci. Paper No. 475, 1923; abb. in Trans. Ill. Eng. Soc. *19*, p. 176, February 1924.

[30] Trans. Ill., Eng. Soc. *19*, p. 607; September 1924.

[31] J.O.S.A. & R.S.I., *7*, p. 1175; December, 1923.
Abst. in Trans. Ill. Eng. Soc. *18*, p. 861; November, 1923.

[32] J.O.S.A. and R.S.I., *7*, p. 243; March, 1923.

of three colored screens must be experimentally determined, but having the three screens makes it possible to match a wider variety of colors, and of course a direct photometric calibration of the colored screens might be made to check the calculated values of transmissions.

A new design of flicker photometer specially planned to fulfill the standard photometric conditions proposed by Ives has been made at the British National Physical Laboratory.[33] A condensed description of characteristic results given by the flicker photometer was published by Ives[34] last year. Extensive papers on the theory and performance of this type of photometer were also published by Ferree and Rand.[35] These papers give very interesting comparisons between observations by equality-of-brightness and by flicker under special conditions. Apparently, however, the methods of observation were chosen to emphasize the effects of growth of sensation, and the results are not directly comparable with those obtained in ordinary photometric work.

Development of physical photometers (i.e. those using some substitute for the eye) has shown no great progress during the past two years. A number of special instruments have been described, mostly for measurements of photographic plates, but all of these have used familiar indicators, including selenium, thalofide and other photoelectric cells, and radiometers of various kinds. As incidental to a discussion of the laws of blackening of photographic plates, Buisson and Fabry[36] have given a valuable general discussion of photometric methods used in photography.

In the field of illumination there have been no specially notable new developments. It is perhaps noteworthy that lamps of various types using glowing gas as the source of light are coming into wider use, chiefly, however, for spectacular lighting or for special purposes where low efficiency is not a serious detriment. Research in illumination is being directed rather largely to studies of the reactions of human beings under lighting of various kinds and amounts, this including both laboratory investigations on individuals and statistical studies of actual production in industrial plants.

[33] J. Guild, Jour. Sci. Inst. *1*, p. 182; March, 1924.
[34] J.O.S.A. & R.S.I., *7*, p. 363; May, 1923.
[35] Trans. Ill. Eng. Soc. *18*, p. 151; February, 1923.
Psychol. Bull. February, 1923.
[36] Revue d'Optique, *3*, p. 1; January, 1924.

4. REFLECTOMETRY

Probably the most important work in reflectometry during the past year was an interlaboratory comparison[37] of reflection-factor specimens ranging in value from 4 to 78 per cent. Three laboratories participated in tests of eleven specimens, part of which were somewhat glossy, others fairly matt. In most cases the agreement was excellent, and within the range of error which one might easily encounter in measurements in a single laboratory.

J. W. T. Walsh[38] has attacked the validity of reflectometry measurements in integrating spheres by methods in which the test surface is viewed directly by the photometer. The basis of his criticism is that, "It is not justifiable to assume that the brightness of a surface which is not perfectly diffusing is the same at all angles of view when the illumination is diffused." His criticism appears to be well grounded and it is entirely possible that under certain conditions measurements made by comparing the test surface with the sphere surface might lead to errors. However, the results of the interlaboratory measurements would seem to indicate that this factor may be of minor importance, since measurements by such a method in one laboratory were in substantial agreement with measurements by a different method in two other laboratories.

G. A. Shook[39] has recently described a combined illuminometer and reflectometer, in which a single lamp is used, thus eliminating the necessity for accurate current control when the instrument is used as a reflectometer. The photometric balance is made by use of a photographic wedge. The reflectometer is a small integrating sphere with a hole in the wall, over which the specimen is placed. By use of the substitution method the brightness of the test specimen, viewed directly by the illuminometer, is compared with the brightness of a standard surface (magnesium carbonate). He assumes that the reflection-factor of the test specimen is equal to the ratio of its brightness to that of the magnesium carbonate when it is substituted for the latter. This assumption is incorrect, however, since the illumination on the specimen under test is a function of the relative area of the sphere wall and opening for the specimen, and of the reflection-factors of sphere wall and of the specimen. Errors of measurement of low reflection-factor specimens may be quite large if this assumption is made.

[37] Trans. Ill. Eng. Soc. *19*, p. 125; February, 1924.
[38] Trans. Ill. Eng. Soc. *18*, p. 475; May, 1923.
[39] J.O.S.A. & R.S.I., *8*, p. 803; June, 1924.

5. COLORIMETRY

The past two years have witnessed a noteworthy enhancement of interest in the scientific and precision aspects of colorimetry. In America, the publication of the Report of the Colorimetry Committee of the Optical Society[40] gave a considerable impetus to the study of this subject and the material compiled in this report is already proving to be of great assistance to those who are attempting to put colorimetric specifications on a sound basis. This publication has not fallen entirely on stony ground; but technologists who have interest in this subject are availing themselves of the aid afforded by the report. Papers by H. E. Ives[41] have also contributed to the same end.

A matter of prime importance which has been receiving considerable attention is the computation of dominant wave length and purity from data on spectral distribution. Extensive and elaborate tests of the agreement of experimental values with values computed from the spectral distribution on the basis of the best data available for the three "sensation curves" or "excitation curves" have been carried out at the Bureau of Standards.[42] While the tests indicate a degree of agreement which is highly gratifying, they also indicate the desirability of a careful redetermination of the "sensation curves." It is understood that such redeterminations are either in course or immediately contemplated by several investigators. The experimental determination of dominant wave length and purity at the Bureau of Standards is now on the footing of routine testing[43] and the results of these tests are being used for such practical purposes as the standardization of railway and traffic signals.

The color grading of incandescent light sources and the various phases of daylight generally has been put on a more definite basis.[44]

An important instrumental development has been the attempts of American instrument makers to produce practical spectrophotometers for measurements of both transmission and reflection in a convenient routine way. While these instruments have not yet reached the full degree of perfection in detail and accessories requisite for all technologic

[40] J. O. S. A. & R. S. I., 6, p. 527, August, 1922.
[41] Jour. Frank. Inst. *195*, p. 23; January 1923.
J.O.S.A. & R.S.I., 7, p. 243; March 1923 and p. 287; April 1923.
[42] Priest, Gibson and Munsell, J.O.S.A. & R.S.I., *8*, p. 28; January, 1924. (See also J.O.S.A. & R.S.I., *10*, p. 291; March, 1925.)
[43] J.O.S.A. & R.S.I., *8*, p. 173; January, 1924.
[44] J.O.S.A. & R.S.I., 7, p. 1175; December, 1923.

work, they are very promising and it is known that the makers are continuing their attempts to develop instruments which will satisfy the demands of technologic colorimetry for the determination of both spectral transmission and spectral reflection.

The publication of the English translation of the second volume of Helmholtz' Physiological Optics will be of great value in putting fundamental theory and data at the disposal of all English workers.

In England, the reports of the National Physical Laboratory, indicate that the problems of colorimetry are receiving considerable attention.

The publication of the second edition of Parson's "Introduction to the Study of Colour Vision" (1924) should be a matter of interest to all who wish to study the basic facts involved in colorimetry.

In Germany, the outstanding feature of current interest in colorimetry appears to be Ostwald's "Farbenlehre" and "Farbenatlas." A voluminous literature has already grown up about this subject and Ostwald's position has been subjected to severe criticism.[45] A study of Ostwald's "Physikalische Farbenlehre" leads the present reviewer to the opinion that Ostwald's technique in physical measurements is very far from being abreast of current knowledge and practice. It will undoubtedly require years of discussion to resolve the differences which have arisen between Ostwald's school and the disciples of Helmholtz.[46] Many of these differences are doubtless due to misunderstanding and confusion of ideas which will require time and the further development of the subject for their settlement.

6. PYROMETRY

Optical pyrometry has grown to mean almost exclusively high temperature measurements with the disappearing filament type of pyrometer. This type is the simplest in construction, affords the greatest accuracy and precision in use and is adapted to a greater diversity of conditions than any other.

Optical pyrometry with its application of the Wien-Planck radiation law to the measurement of temperature has been regarded in the past

[45] Kohlrausch: Phys. ZS. *21*, p. 396, p. 423 and p. 473; 1920,
Van Arsdel, Paper, p. 113; April 18, 1923.
[46] Kohlrausch: loc. cit. Van Arsdel: loc. cit.
Meissner. Phys. ZS., *22*, pp. 268 and 641; 1921,
24, p. 213; 1923.
Seitz: Phys. ZS. *23*, p. 297; 1922.
Seitz: Zeits. fur Sinnesphysiol. *54*, p. 146; 1922.
Ostwald: Deutsche med. Wochenschr. *48*, p. 1236; 1922.
Schaller: Augsburger Postzeitung; March 7, 1921 et seq.

and is still regarded by many as a purely secondary means, lacking a firm theoretical foundation. This viewpoint is taken by Day and Sosman,[47] but there seems to be no justification for a complete disregard of the vast amount of experimental confirmation of the quantum theory and the consistency of the experimental values of C_2, h, σ and other constants with the theoretical relations connecting them.

It seems reasonable, therefore, to take the best mean value for constants such as C_2 and the palladium melting point by including values obtained with the aid of Planck's law, instead of adhering solely to determinations derived from the gas thermometer. With such a viewpoint, Waidner, Mueller and Foote[48] obtained a mean value of 1552°C for the melting point of palladium on the basis of $C_2 = 1.435$ cm deg. Using the present value 1.432 for C_2 the palladium point becomes 1553°C, which is within the error of ±3° allowed by Day and Sosman from their own value of 1550°. At the Bureau of Standards Fairchild has recently obtained the value 1553°C, using $C_2 = 1.432$, with the optical pyrometer, and the uncertainty of this determination is not more than 3°. Of seven values by different authors used in obtaining a mean of 1553°, the average departure from this mean is 2°. The melting point of palladium may properly be stated as 1553°, with an uncertainty of not more than 2°.

It is interesting to note here that Ives[26] has derived a value for the melting point of platinum which must be given some weight. Using his value of 55.4 candles per cm² for the brightness of a black body at the melting point of platinum and interpolating with this value in the data of Hyde, Forsythe and Cady[27] on the brightness of a black body at various temperatures, Ives obtained the value 2035°K or 1762°C for the melting point of platinum if $C_2 = 1.435$ as used by Hyde, Forsythe and Cady. This value is raised to about 1764°C if $C_2 = 1.432$.

A preliminary report from the Reichsanstalt by Hoffman[49] gives a value of 1772° for $C_2 = 1.430$ extrapolating from 1063°C. Apparently a spectrophotometer was used to sight into an iridium wound magnesia furnace. Probably the "wire method" was followed, with a bit of platinum wire fused into the hot junction of an iridium-ruthenium thermocouple. Until a complete report is obtained no criticisms can be made. The value 1772°C seems too high at present.

[47] Dictionary of Applied Physics, *1*, p. 862; 1922.

[48] Bull. 153, Amer. Inst. Min. Met. Engrs. p. 2051, Sept. 1919 and Pyrometry Symposium, published in book form, p. 46; 1920.

[49] ZS. f. Instrumentenk. *43*, p. 96; 1923 (See also later report, ref. No. 28).

The melting point of carbon or graphite is given by Fajans and Ryschkewitsch[50] as 3800°K ± 100°. If carbon has a melting point these authors apparently observed melting taking place. The temperature at which a rod melted through was measured with a Holborn-Kurlbaum pyrometer. The authors give insufficient data and description of the method of calibrating the pyrometer to permit much weight being given to the result obtained.

Worthing[51] has made recent measurements on the emissivity of tungsten at 300°K and 1330°K by a reflection method and has published a table of emissivity versus temperature, based on $C_2 = 1.435$ and T (gold) = 1336°K.

For the melting point of tungsten Worthing obtains as the weighted mean of nine groups of observers, 3645°K for $C_2 = 1.435$ and 3668°K for $C_2 = 1.430$. On the basis of $C_2 = 1.432$ the melting point is 3654°K or 3381°C, which can be rounded to 3380°C. Worthing concludes that the melting point is known to ±20°.

Development of optical pyrometry is indicated by an article on disappearance of the filament and diffraction effects by Fairchild and Hoover[52] who state that, "The precision, accuracy and wide range of usefulness of the disappearing-filament optical pyrometer depend on perfect disappearance of the filament with high resolving power and magnification by the eyepiece." The paper includes an extensive study of diffraction effects, and discusses in detail the conditions under which complete disappearance of the filament can be obtained.

As an indication of the precision which is attainable with the optical pyrometer when it is properly designed, Fairchild and Hoover st ate that a precision of 0.2°C or better at the melting point of gold can be obtained.

The development of this instrument as one of precision is well illustrated by the data presented by Forsythe[53] in "An Intercomparison of Temperature Scales." This paper deals with a comparison of the high-temperature scales in use at the Bureau of Standards, the General Electric Company's Research Laboratory at Schenectady, the Physical Laboratory at the University of Wisconsin, the National Physical Laboratory of England, and the Nela Research Laboratory at Cleveland.

[50] Naturwissensch *12*, p. 304, No. 16; 1924.
[51] ZS. f. Physik, *22*, p. 9; 1924.
[52] J.O.S.A. & R.S.I., *7*, p. 543; July, 1923.
[53] Astrophys. Jour. *58*, p. 294; December, 1923.

"The agreement found is very good, the maximum difference being only a few degrees for any point of the entire range from 1400°K to 2700°K."

An extensive and intricate study of the "Use of Absorption Glasses in Optical-Pyrometric Measurements" has been made by F. Hoffman,[54] but unfortunately this paper has made the subject appear more complicated than it actually is. In particular, Hoffmann's discussion of "effective wave length" of filters is erroneous, and does not take account of some important relations published in this country. This paper clearly demonstrates the need of a résumé of the principles of optical pyrometry and particularly of those referring to the use of absorption devices for measuring higher temperatures and the temperatures of non-black bodies, since there is no published article which gives all the principles and data and methods of computation, in such form that they may be readily understood.

[54] ZS. f. Physik. *17*, p. 1; 1923.

Helmholtz's Treatise on Physiological Optics.—Translated from the Third German Edition. Edited by James P. C. Southall, Professor of Physics at Columbia University. Volume II, The Sensations of Vision. VIII+480 pages. Published by the Optical Society of America, 1924.

The first volume of the English translation of Helmholtz's Treatise on Physiological Optics, which was reviewed in this journal, January 1925, is particularly of interest to the Ophthalmologist and Physicist, since it deals with the anatomical and physical considerations that are involved in the construction and motions of the eye, and in the formation of images upon the retina.

The second volume which has now been issued from the press contains an elaborate discussion of the phenomena of color vision which are essential to an understanding of the theory of its perception. As color vision at the present time is probably attracting a greater degree of attention both from the theoretical and practical points of view than ever before in its long history, the appearance of this translation could scarcely have been more opportune. From the editorial standpoint the second volume maintains the high standard set by the literary excellence of the first. The reviewer has again compared all the mathematical equations and tables of constants with the German text and found only a couple of quite obvious and unimportant misprints. The verbal misprints noticed were communicated to the editor. From these considerations and the evident carefulness of the translators, the English text may be consulted with the greatest confidence as a faithful

representation of the original work. Professor Southall and his collaborators are entitled to regard the successful accomplishment of the translation of these two volumes with the greatest satisfaction, and they may be fully assured that their labors will receive the grateful appreciation of all present and future workers in the broad field of vision.

In this volume the supplements by Professors Nagel and von Kries, which were prepared for the Third German Edition, and the extensive notes by the same writers scattered throughout the text, some of which were written especially for the English translation, constitute about one third of the whole. These deal with subjects scantily noticed by Helmholtz, or which have been developed since his time, such as the important subjects of Adaptation, Twilight Vision and the Duplicity theory, and a comparison of Normal and Anomalous color systems. The volume concludes with a critical review of the chief theories of color vision by von Kries, and a supplement on the same subject by Mrs. Christine Ladd-Franklin, which contains also a brief exposition of her theory. The volume throughout is enriched also with many footnotes by the editor and his associates in the work of translation. A page of corrigenda in Volume I is also included.

From this summary it is apparent that the volume is not only Helmholtz's treatise on Color Vision, but a broad exposition by other great authorities of the fundamental facts and principles of the subject and their theoretical significance.

It is probable that in no branch of science have so many hypotheses been devised as in color vision. Their number can scarcely be less than four score, and nearly every year brings an addition to the already long list. From this condition it may be inferred that great difficulty has been experienced in completely harmonizing on any simple basis the extraordinary diversity of facts that must be explained consistently with each other.

The hypothesis of Young, that three primary sensations were necessary and sufficient to explain the varied phenomena of color, has been the inspiration of all subsequent theories with but few exceptions. Helmholtz himself acknowledges his great indebtedness to Young as the following quotation from his lecture on "The Recent Progress of the Theory of Vision" (1868) clearly shows: "The theory of colors, with all these marvellous and complicated relations, was a riddle which Goethe in vain attempted to solve; nor were we physiologists and physicists more successful. I include myself in the number, for I long toiled at the task, without getting any nearer my object, until I at last discovered that a wonderfully simple solution had been discovered at the beginning of this century, and had been in print ever since for anyone to read who chose. This solution was found and published by the same Thomas Young who first showed the right method of arriving at the interpretation of Egyptian hieroglyphics. He was one of the most acute men who ever lived, but had the misfortune to be too far in advance of his con-

temporaries. They looked on him with astonishment, but could not follow his bold speculations, and thus a mass of his most important thoughts remained buried and forgotten in the 'Transactions of the Royal Society,' until a later generation by slow degrees arrived at the rediscovery of his discoveries, and came to appreciate the force of his arguments and the accuracy of his conclusions."

The justification for issuing the present volume can scarcely consist in elaborating in detail the Young-Helmholtz theory merely for the purpose of exhibiting it as a colossal ruin. On the contrary, it is rather to display a massive structure that has in the main stood the test of time and of a multitude of assaults, for the principle of trichromasy in color theory has so successfully maintained its position that the majority of rival theories in one form or another have essentially the same basis. That the foundations of the theory are still intact von Kries most emphatically asserts. "Even at the present time the theory of Helmholtz is thoroughly justified as to its fundamental conceptions; it is in close agreement with the facts, and as an hypothesis it is qualified to explain a very large mass of actual phenomena. . . . . Fundamentally, the Helmholtz theory was simply the expression of a direct fact of observation, namely, that the resultant of all the various light stimuli so far as sensations are concerned, can be completely represented as a function of three variables. It is idle to try to explain this fact except on the assumption that the result of stimulation also can be represented completely as a function of three variables. It is this assumption that is the fundamental conception of the Helmholtz theory. The justification for it has been shown most of all by the huge difficulties that are encountered by every theory that has been developed on the assumption of a greater number of independent valences." (Page 426.)

From such considerations as these it is difficult to understand the judgment of Dr. Cattell in a review of the Second Edition that "if the theory were to be proposed at this time (1898) it would not have a single adherent," or that of Professor William James that "Helmholtz is in the science of color more eminent for his experimental work than for his theoretical contributions." (Page 458.)

In regard to the explanation of simultaneous contrast the theory of Helmholtz has undoubtedly suffered chiefly because in this particular he abandoned the more secure foundations of physiology for the less tangible principles of psychology. His hypothesis of illusory judgments has been attacked by almost all theorists and from many standpoints shown to be fallacious, though none of the rival theories designed to replace it have had any greater success. Hering himself believed that stimulated areas of the retina reciprocally influenced each other so causing contrast, but he was unable to suggest any means by which such action was accomplished. It is interesting to read in a note prepared by von Kries for this translation that, though also unable to offer proof, he endorses the suggestion of Hering as far as the explanation of luminosity contrast is concerned. "However," he says, "it is another question as

to whether this is the sole cause of contrast phenomena or whether conditions of another sort are likewise involved, especially conditions that are identical with, or at any rate not very far removed from those which Helmholtz inferred. At present the latter must be considered as being probably the case. All the facts that led Helmholtz at the time to attribute these phenomena to mental illusions and that were adduced by him in support of this hypothesis are in its favor. Moreover, some very searching experiments on this point have been made very recently by Jaensch and his pupils, the conclusion being that many, in fact most, contrast phenomena are not to be accounted for by a reciprocal physiological action such as Hering assumed." (Page 300.)

In regard to the interaction of retinal areas the reviewer has very recently proved experimentally that sensory reflex actions are operative in the visual apparatus by means of which stimulation of one retinal area affects the adjoining regions, and also the other eye, in such a manner as adequately to account for the phenomena of both luminosity and color contrast.

In the critical supplement on the Theories of Vision, von Kries discusses at length the modifications of the Young-Helmholtz theory which are necessary to bring it into harmony with the major phenomena of vision such as those connected with twilight vision, color blindness and anomalous trichromatism. Other theories are also critically reviewed, the chief of which is necessarily Hering's. The conclusions of von Kries, however, are strongly adverse not only to Hering's form of the theory but also to the modifications which have been made to it by Müller and Tschermak. While these criticisms are quite justifiable, yet the complete rejection of Hering's theory is far too sweeping. For that distinguished investigator introduced at least two ideas which are essential in modern color theory. The first is the reciprocal action of retinal areas upon each other, which has just been referred to, and the second is the recognition of opponent visual processes. These also the reviewer has experimentally proved to exist, but with the important difference that the antagonistic actions are not between certain pairs of colors, but that they consist of opponent nerve actions consequent upon stimulation with all colors.

For incisive criticism of these and of many other modern theories of color vision this supplement is of conspicious interest and value. The critical reader, however, will notice in all the supplementary sections, including that of Mrs. Ladd-Franklin, the paucity of references to work done by British and American investigators. Were it nor for the numerous footnote references, and the extensive and valuable bibliography covering the years 1911-24, prepared especially for this edition, the reader might unconsciously infer that for many years color vision has been almost exclusively a German science.

The concluding supplement in the volume is a discussion by Mrs. Ladd-Franklin on The Nature of the Color Sensations, in which is included a brief exposition of her theory, written in her vivacious style.

The first part is controversial in character and is directed impartially against both Young-Helmholtz and Hering theories. Physicists in general receive a generous share of her criticism, and four are singled out for special disfavor because their use of the word, color, strictly implies an objective reality in that quality of sensation. Helmholtz falls under the same condemnation, but is excused as with him it is "a mere momentary inadvertence," though as a matter of fact his inadvertence extended through two editions of his work separated by twenty years. It is again alleged (page 458) that the physicist finds it impossible "to realize that when once light has struck the retina wave lengths cease to exist—that their place is taken by three initial chemical products and mixtures of them." Notwithstanding this indictment, few will imagine that the physicists who write on color are in the least degree ignorant of this elementary fact. But even so rigorous a terminological accuratist—if one may coin a word—as Mrs. Ladd-Franklin herself quite naturally glides into precisely the same kind of error when she says "that all the colors of the spectrum can be matched by physical mixtures of red, green and blue lights." (Page 459.) In describing some of the processes of color vision which involve so many diverse sciences, it seems to be practically impossible to devise concise and at the same time flawless modes of expression by which cumbrous circumlocution may be avoided. Astronomers also use scientifically inaccurate terms, such as sunrise and sunset, and are blameless.

The theory of Mrs. Ladd-Franklin is essentially a theory of visual mechanism. It is based upon several unproved and unprovable assumptions. "It is assumed that there is a light-sensitive substance in the rods which gives off, under the influence of light, a reaction-product which is the basis of the primitive sensation of whiteness. In the cones, in the next higher stage of development of the color sense (the yellow and blue vision of the bees, and of our own mid-periphery), this same light-sensitive substance has become by a simple molecular rearrangement, more specific in its response to light, and in such a way that the two ends of the spectrum act separately to produce nerve-excitant substances which, however, when they are produced both at once, unite chemically to form the "white" nerve-excitant out of which they were developed. In the third and final stage the "yellow" nerve-excitant has again undergone a development in the direction of greater specificity, and red and green are acquired." (Page 464.) We have, however, no assured knowledge that whiteness is an independent sensation apart from the color sensations. (Page 61.) The three facts, that a color sensation is never saturated, that two complementary colors will produce a sensation of whiteness, and that brief flashes of white light may cause various color sensations instead of white, ought at least to prevent dogmatism on this phase of the subject.

The introduction of the alleged color vision of insects and animals into theories of human color vision has at best a precarious value. When it is remembered how difficult it is to determine the actual color percep-

tions of intelligent people with defective color vision, one can scarcely be justified in confidently assigning to bees a limited yellow-blue vision. As these insects have inhabited the earth as long as man, and have passed through vastly more generations in all of which their environment has been notably characterized by variegated color, their vision in regard to color might well be supposed to be even more highly developed than our own. The well-known predilection for red clover which bees exhibit, in itself would suggest the exercise of a high degree of caution in estimating their color vision to be restricted to yellow and blue.

As Mrs. Ladd-Franklin lays great stress upon the evolutionary character of her theory, it may be recalled that Edridge-Green also has devised an evolutionary theory which differs from it in almost every particular. He, too, makes the broad assertion that every known fact of color vision is in perfect accord with his theory. The same claim for her theory is made by Mrs. Ladd-Franklin in a very brief paragraph which in one stroke cuts the Gordian Knot of every difficulty that color vision possesses: "The accessory phenomena of color are also given a perfectly simple and satisfactory explanation in this theory." (Page 465.) It would be of interest to have on this theory an explanation of the process by which the accessory phenomena of binocular contrast, for example, are brought about.

With necessarily an imperfect knowledge of the facts of color perception, or, indeed, of any branch of science, it is dangerous to claim for a theory the fatal characteristic of perfection. "But it is well known," says Mrs. Ladd-Franklin, "that successful theories of complicated occurrences in nature are not only intellectually satisfying but also most important as guides to further investigation." (Page 457.) Judged by this test the theory of Young and Helmholtz has rendered by far the greatest service of all. The two evolutionary theories, if either were correct, from their very nature should be expected to excel all others in suggesting new lines of research, while on the contrary both have been conspicuously lacking in this power.

The volume so abounds with suggestive material that a review might be almost indefinitely extended. Let it suffice to say in conclusion that in this work we have the finely ordered presentation of the phenomena of color vision by Helmholtz, and critical and expository supplements by notable investigators in the same field, all presented in a translation of uniform literary excellence, and set forth in a volume which is an elegant example of the printer's art.

FRANK ALLEN

INSTRUMENT SECTION

THE THEORY OF THE SINGLE FIBER ELECTROSCOPE AND A NEW DESIGN OF THE INSTRUMENT*

By W. F. G. Swann

ABSTRACT

If k is the mechanical restoring force per unit displacement of the fiber, the actual restoring force per unit displacement may readily be shown to be $k-CX^2$, where C is the capacity of the fiber and X the field between the plates. The deflecting force due to a change of potential δV in the fiber is $CX\delta V$, so that the sensitivity S is $S=CX/(k-CX^2)$. If k is less than CX^2 the instrument is unstable, and the fiber flies to either one plate or the other. The sensitivity becomes very large if $k-CX^2$ is small; and in obtaining high sensitivity from the instrument, this condition is usually satisfied, even though the fact may not be realized. It is a bad condition since it results in large variations in S for small variations in either k or X. If we desire to restrict ourselves to the case $k-CX^2=k/2$, so that we draw upon the term CX^2 to the extent of no more than doubling the sensitivity which we should obtain in its absence, S reduces to the very simple expression $S=1/X$ which, under a magnification of 100, gives a deflection of 1000 mm per volt for a field of one volt per cm between the plates. High sensitivity consistent with great constancy necessitates a small field between the plates and correspondingly small value of k. An instrument meeting the required conditions is described. As an illustration of the performance of the electroscope one experiment gave, with a magnification of 600, and a potential difference of 6 volts between plates, a sensitivity of 3500 eyepiece divisions per volt and a period so short that the fiber assumed a steady reading in less than 3/4 second after application of the potential. Moreover, the linearity of the calibration curve over the whole range of the eyepiece scale was so perfect that departures from linearity could not be observed within the limits of accuracy of the readings.

The single fiber electroscope in its usual form comprises a conducting quartz fiber fastened to a lug at the top and a quartz bow at the bottom, and mounted between two parallel plates to which can be applied potentials $+V$ and $-V$. The variations of potential to be measured are applied to the fiber in the customary manner of using the instrument.

Those who have had experience with the instrument will be familiar with the fact that, if the field between the plates is made large or the tension on the fiber small so as to approximate to high sensitivity, the fiber shows a tendency to fly to one side or the other of its zero position and stay there, even when kept at the potential of the case. In other words the system is unstable. At the limit of stability the

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instrument is theoretically infinitely sensitive, but attempt to use it at a sensitivity of 1000 eyepiece divisions per volt, even with the highest practicable magnification, results in a condition where the sensitivity varies very greatly over the scale range, and varies, moreover, greatly with the time, on account of temperature changes, fluctuation of battery potentials and the like. While the writer has succeeded in using the ordinary instrument at a sensitivity of 2000 divisions per volt, his experience is that the ease and reliability of the ordinary Dolezalek quadrant electrometer at 1000 divisions per volt on a scale 1 meter distant is about the equivalent of the ease and reliability of the fiber electrometer at 10 divisions per volt with the usual magnification. The purpose of this paper is to give the reasons for the peculiarities of behavior of the single fiber instrument and to describe an instrument designed in such a manner as to take full account of these considerations.

THEORETICAL

Suppose X is the field between the plates, and let us in the first instance consider a case where all parts of the fiber between the plates suffer the same displacement when the fiber is deflected from its zero position. Let C be the capacity of the portion of the fiber between the plates, and k the mechanical restoring force per unit displacement of the fiber from its zero position. If the fiber be kept at the potential of the case and be displaced by some means through a distance δx, it will experience a mechanical restoring force $k\delta x$, and an electrical force tending to increase δx. For, since the fiber is brought to a place where the potential due to the field is $X\delta x$ it will receive a charge $-CX\delta x$, and there will be a force $CX^2\delta x$[1] tending to increase the displacement.[2] The net restoring force per unit displacement is then $k-CX^2$. If the

[1] The attraction of the fiber for its images in the plates is negligible, and is here neglected.

[2] A more rigorous presentation of the matter is the following. Suppose q_{11}, q_{12}, q_{13} are the coefficient of capacity of the fiber and its coefficients of induction in relation to the plates in the displaced position represented by δx. Then, if the fiber be displaced subject to the condition that its charge remains zero, its potential will change by $X\delta x$ and we shall have $q_{12}V_2+q_{12}V_3+q_{11}X\delta x=0$ where V_2 and V_3 refer to the potentials of the plates.

If, keeping the fiber in the same position, it be brought to zero potential, and if Q is the charge which comes to it under these conditions $q_{12}V_2+q_{13}V_3+q_{11}\times 0=Q$. Hence, from these two equations

$$Q=-q_{11}X\delta x$$

agreeing with the result in the text when C is written for q_{11}. In practice it is, of course, sufficient to regard C as pertaining to the fiber for its undeflected position.

fiber be displaced through the distance δx by the application of a potential δv, the relation between δv and δx will be

$$CX\delta v = (k - CX^2)\delta x \tag{1}$$

Hence, the sensitivity, which we shall define as $S = \delta x/\delta v$ is given by

$$S = \frac{CX}{k - CX^2} \tag{2}$$

Thus, the instrument becomes unstable if $k \leq CX^2$, and the smaller $k - CX^2$ the greater the sensitivity.

Now, even though one who is using the instrument may be unaware of the fact, he ordinarily attains high sensitivity by making CX^2 very nearly equal to k. In other words, the actual restoring force per unit displacement is the difference of two quantities k and CX^2, each of which is many times that difference. This situation is obviously bad, for a small variation of either of these terms produces a very much more important effect on the difference. Moreover, departure from linearity in the relation between either the mechanical restoring force $k\delta x$ and the displacement, or the electrical "negative" restoring force $CX^2\delta x$ and the displacement results in a greatly enhanced departure from linearity in the relation between the resultant restoring force and the displacement.

Suppose we call the legitimate sensitivity the sensitivity we should have if the restoring force were $k\delta x$ alone, and the actual sensitivity the sensitivity corresponding to the actual restoring force $(k - CX^2)\delta x$. Then, if

$$k - CX^2 = k/n \tag{3}$$

so that the actual restoring force is only one n^{th} of the mechanical restoring force and the actual sensitivity is therefore n times the legitimate sensitivity, we shall have, from (3),

$$k - CX^2 = \frac{CX^2}{n-1}$$

so that

$$S = \frac{n-1}{X} \tag{4}$$

a remarkably simple result, and one showing that for a given n, S actually increases with diminution of the field between the plates.

Suppose the quantity X suffers a change δX. Then, by logarithmic differentiation, and using 2,

$$\left(\frac{\delta S}{S}\right)_{k=\text{const.}} = \frac{\delta X}{X} + \frac{2CX\delta X}{k-CX^2} = \frac{\delta X}{X} + 2S\delta X$$

Using 4,

$$\left(\frac{\delta S}{S}\right)_{k=\text{const.}} = (2n-1)\frac{\delta X}{X} \tag{5}$$

Again, if k varies by δk, we obtain by logarithmic differentiation, using 2,

$$\left(\frac{\delta S}{S}\right)_{X=\text{const.}} = -\frac{\delta k}{k-CX^2} = -n\frac{\delta k}{k} \tag{6}$$

We thus see that if the actual sensitivity is n times what we have called the legitimate sensitivity, a one per cent variation in k produces n per cent variation in the actual sensitivity S, and one per cent variation in X produces $2n-1$ per cent change in S.

n is thus a quantity whose magnitude gives an indication of the degree of constancy necessary in X and k for a given percentage constancy of the sensitivity. It, or rather its reciprocal, is a quantity which may be taken to specify the "reliability" of the instrument, the reliability being large when n is small. In comparing sensitivities it is convenient to compare them for a fixed value of n; and, as we have already shown in equation 4, for a given n (a given degree of reliability), the sensitivity is inversely proportional to the field, and is independent of everything but the field.

If $n=2$, so that the mechanical restoring force is only twice the actual restoring force, equation 4 assumes the very simple form

$$S=\frac{1}{X}$$

so that, for a field of 1 volt per centimeter between the plates, the sensitivity should be 1 cm per volt, i. e., 10^{-3} cm, per millivolt; or, under a magnification of 100, it should amount to 1 millimeter per millivolt.

It may seem paradoxial that we have arrived at the conclusion that high sensitivity necessitates a *small* field between the plates, but it must be remembered that we are speaking of a constant degree of reliability,

and so a constant n; and, in order to keep this degree of reliability constant when we reduce the field, we must appropriately reduce the mechanical restoring force. Thus, in the example quoted, since $X = 1/300$, we see from 3 that, if $n = 2$,

$$k = \frac{2C}{9 \times 10^4}$$

Now it turns out in practice that, in order to reduce k to such an extent as to make possible the use of fields of the order 1 volt/cm between the plates, it is necessary to dispense entirely with the quartz bow which usually functions for providing the mechanical restoring force; for, the weight of the fiber itself becomes sufficient to provide all the restoring force that is necessary. Of course C diminishes with diminution of diameter of the fiber, but only logarithmically, so that reduction of the diameter results in a reduction in k/CX^2 for a given X. Of course, in order that the period may not be unduly long, it is necessary to work with a short fiber.

NUMERICAL EXAMPLES

We shall seek the conditions necessary to give a sensitivity of 1000 mm per volt with a magnification of 100 (the microscope being focused on the lower end of the fiber), a condition of working in which the actual sensitivity is only twice the legitimate sensitivity, and a period of about one third of a second as unaffected by electric influence, or $\sqrt{2}$ times this amount under the actual working conditions.

In order to avoid unnecessary complexity in the calculations it will be necessary to make certain approximations which do not affect the orders of magnitude of quantities representing all that we are interested in.

In the first place, since the smallness of the mechanical forces necessary requires that the fiber swing as a sort of pendulum, it will not be correct to assume that all parts of it suffer the same linear displacement unless we are willing to have the point of suspension unduly far above the region of the field X, a condition which would unnecessarily increase the period for a given sensitivity. Again, the distribution of the charge over the fiber will be rather complicated, and the fiber will not remain truly straight during the deflection. We shall suppose that the point of suspension of the fiber lies in the field and that the fiber itself moves as a rigid rod freely suspended at the top. If m is the mass of the fiber, and l its length, the mechanical restoring couple for a deflec-

tion $\delta\theta$ is $mg\ l\delta\theta/2$. The charge which would come to the fiber if it were at zero potential in its deflected position is not the same as if the whole fiber were displaced through an amount $l\delta\theta/2$ because the portions toward the end acquire more charge per unit displacement than corresponding portions elsewhere. Again, the effective position of the charge is nearer to the end than the middle of the fiber. We shall assume that, as regards the charge acquired, the effect is the same as if the whole fiber were displaced through an amount $l\delta\theta/\epsilon$, and we shall suppose that the effective position of the charge is at a distance l/ϵ from the point of suspension, ϵ being a quantity between one and two. The assumption that the same ϵ will serve for both of these cases is probably not serious as regards the orders of magnitude involved. Then the electrical "negative" restoring couple which is the representative of the second term on the right hand side of 1 is equal to $-l^2CX^2\delta\theta/\epsilon^2$. If we also assume that the charge which comes to the fiber as a result of its being raised through the potential δv is situated at a distance l/ϵ from the end, the couple on account of this charge, and which corresponds to the left hand side of 1 is $lCX\delta v/\epsilon$. Thus

$$\left(\frac{mgl}{2} - \frac{CX^2l^2}{\epsilon^2}\right)\delta\theta = \frac{l}{\epsilon}CX\delta v$$

If δx is the displacement of the end of the fiber, $\delta\theta = \delta x/l$. Hence

$$\frac{\delta x}{\delta v} = \frac{\epsilon CX}{\frac{mg\epsilon^2}{2l} - CX^2} \tag{7}$$

so that, for a condition where the actual restoring couple is half the mechanical restoring couple, i. e., for

$$\frac{mg}{2l} - \frac{CX^2}{\epsilon^2} = \frac{mg}{4l} \tag{8}$$

we have

$$\frac{\delta x}{\delta v} = \frac{\epsilon}{X}$$

Since, owing to the condition of our problem

$$\delta x = 10^{-2}\text{ cm} = 10^{-3}\text{ mm, for } \delta v = 10^{-3}/300\text{ esu,}$$

we find, on substituting in 8,

$$X = \epsilon/300\text{ esu.} = \text{between one and two volts per cm.}$$

Treating the fiber as a thin rod of length l, its moment of inertia is $ml^2/3$. The mechanical couple per unit angular displacement is $mgl/2$, and the actual couple per unit angular displacement for the present case is $mgl/4$. Thus the period T, unmodified by electrical forces is

$$T = 2\pi\sqrt{\frac{4ml^2}{3mgl}} = 0.22\sqrt{l} \tag{9}$$

so that if l is of the order 2.5 cm, the period will be about 0.35 sec. Since in the present instance the actual restoring couple is half the mechanical restoring couple, the actual period will be $\sqrt{2}$ times the above value.

Equation 8, for $X = \epsilon/300$ esu as above gives

$$mg = \frac{4Cl}{9\times 10^4}$$

so that if ρ is the density and r the radius of the fiber,

$$\pi r^2 \rho g = \frac{4C}{9\times 10^4}$$

and

$$r = \left(\frac{4C}{9\pi\rho g\times 10^4}\right)^{1/2}. \tag{10}$$

We have seen that, in order to obtain a period of 0.35 sec. as measured when undisturbed by electrical forces, we must have $l = 2.5$ cm. The capacity of the fiber is approximately $l/(2\log_e 2b/r)$ where r is its radius, and $2b$ the distance between the plates.[3] If $2b = 0.5$ cm, $r = 0.3\times 10^{-4}$ cm and $l = 2.5$ cm, $C = 0.16$ cm.

Putting $\rho = 2.2$ for fused quartz, equation (10) gives $r = 0.29\times 10^{-4}$ cm. It is of course evident that, since $2b/r$ is of the order 10^4, its logarithm would not be appreciably affected to the limits of accuracy to which we are working by recalculating it on the basis $r = 0.3\times 10^{-4}$ instead of 0.29×10^{-4}. It then appears that, with a fiber 2.5 cm long and 0.6 micron in diameter, and a field of about 2 volts per centimeter between the plates we should obtain a sensitivity of 1000 divisions per

[3] This value of the capacity may readily be obtained as a very close approximation in the case of infinite plates by considering the fiber and its various images in the plates and then paring them off suitably in such a way as to leave only the fiber itself and its image in one of the plates.

volt under a magnification of 100. Moreover, the condition of wor would be such that the actual restoring couple for a given deflection was as great as half the mechanical restoring couple, so that the actual period would be only $\sqrt{2}$ times the period as uninfluenced by electrical forces, which period would in this case amount to only 0.35 second. Of course, in order to secure the advantages resulting from the short period, it is necessary to work with the fiber in a vacuum so as to reduce damping.

Since the sensitivity for a given degree of reliability as determined by n is proportional to X, and since, from (8), $\sqrt{m/l}$ is proportional to X, the sensitivity is proportional to $\sqrt{l/m}$; or, since for a fixed period T, l is given by (9), the sensitivity for a given T is inversely proportional to $\sqrt{m}$, and so to r.

In general, the sensitivity $S=\delta x/\delta v$ is given by (7), so that, since $C=l/(2\log_e 2b/r)$ approximately,

$$S=\frac{\epsilon}{X\left[\frac{\pi r^2\rho g\epsilon^2}{lX^2}\log_e\frac{2b}{r}-1\right]} \tag{11}$$

The quantity which we have called the legitimate sensitivity (say S_0) is obtainable from (7) by omitting the term CX^2. Thus

$$\frac{S}{S_0}=\frac{1}{1-\frac{lX^2}{\pi r^2\rho g\epsilon^2\log_e\frac{2b}{r}}} \tag{12}$$

A word must be added regarding the bending couple due to the elasticity of the fiber, which has been neglected in the above. For a circular cylinder of length, l, the couple for a displacement δx of one end, the other end being fixed may readily be seen to be given by F_r, where

$$F_r=\frac{3\pi r^4 Y}{4l^2}\delta x$$

Y being Young's modulus for quartz. The only couple considered in our calculation is the couple F_g due to gravity which is

$$F_g=\frac{\pi r^2\rho l}{2}g\cdot\frac{l\delta x}{l}$$

unite,

$$\frac{F_r}{F_\theta} = \frac{3}{2}\frac{r^2 Y}{l^3 g\rho}$$

Putting $Y = 5\times 10^{11}$ dynes/cm^2 for quartz, $\rho = 2.2$ and $l = 2.5$, we find, for $r = 0.3\times 10^{-4}$

$$\frac{F_r}{F_\theta} = 0.02 \qquad (13)$$

which is sufficiently small to justify our neglect of F_r. The importance of F_r mounts rapidly with increase in the radius of the fiber, however; and, for these cases causes the sensitivity to be less than that corresponding to the above conditions. Thus, for a fiber 3 microns in radius, F_r/F_θ would be 100 times as large as the value given by (13), and would be equal to 2.

DESCRIPTION AND PERFORMANCE OF AN INSTRUMENT CONSTRUCTED ON THE FOREGOING LINES

We have seen that the most satisfactory conditions of working for high sensitivity are to be obtained with a short, and very fine conducting fiber suspended at one end. In the instrument constructed for the purpose of illustrating the foregoing principles, the two insulated plates *PP*, Fig. 1, were rectangular in shape, 2 cm long, and 9 mm wide. The fiber *F* was about 1 micron in diameter and about 2.2 cm long and was coated with platinum by evaporating from a hot wire. The diameter of the electroscope case *E* was 3 cm and its height 3 cm. In order to mount the fiber a special device was necessary, since a fine fiber with one end free floats around in a manner difficult to control. This device consisted of a square frame, *ABCD*, with a rod *GH* which was insulated from the remainder. The lower part of the case of the electroscope could be slid down this rod *GH* leaving only the lid with the lug *KL* fastened at the top. Under these conditions the frame was laid with its plane horizontal and a platinized quartz fiber of length equal to *HL* was placed on a glass plate situated immediately below and parallel to *HL*. One end of this fiber was then soldered leaving the remainder of the fiber lying on the plate. By means of a transformer a potential difference of 1000 volts was then placed between the part *GH* and the remainder of the frame. On carefully raising the fiber from the plate by means of a needle, it suddenly shot out in a horizontal direction pointing toward *H*, its free end being now almost a millimeter from *H*.

Under these conditions the fiber acted almost like a rigid rod, and the electroscope could be slid up into position, assembled and carried about without the slightest danger of losing the fiber. It is always possible to remove the frame after the electroscope is assembled and set up for permanent use; but, in doing so care must be taken to keep the fiber connected to the case during the operations. Also, it is convenient to arrange so that the insulated part *GH* remains permanently. Then,

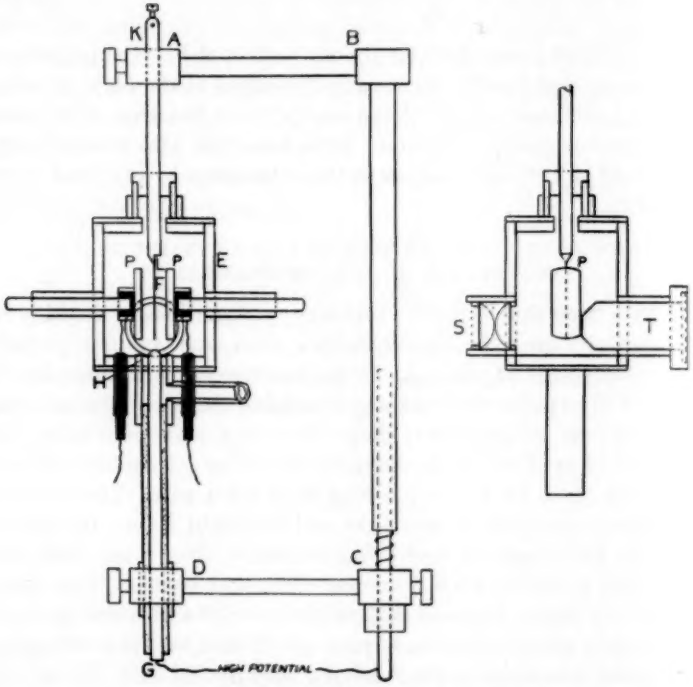

FIG. 1. *Instrument used in experiments described.*

whenever one wishes to perform any violent manipulations with the electroscope such as might endanger the fiber, he can always put a potential on *GH* and so hold the fiber firmly in position. With a fiber a micron in diameter, the convection currents in the case are sufficient to cause trouble at atmospheric pressure (with no potential on *GH*) by causing the fiber to float up and touch one of the plates. All trouble of this kind vanishes when the electroscope case is evacuated, which evacuation is of course carried out with the potential on *GH*, and the

instrument here described was left standing for nearly three months with the fiber in position with zero potential on *GH*, and with no better precautions to preserve the vacuum than a rubber tube joining the instrument to a stopcock. Of course, before using the electroscope for exact measurements the vacuum should be improved to the desired amount which is determined by the condition that the fiber should be just about dead beat.

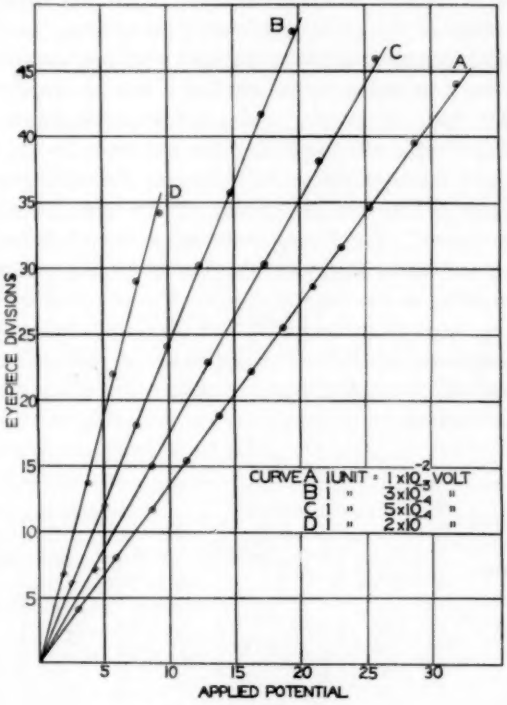

FIG. 2. *Calibration Curves.*

In order to insure proper optical illumination a condenser *S* was used. Since in view of the vacuum it was inconvenient to focus the objective finely, the plan adopted was to use the primary objective *T* to form an enlarged image outside the instrument and then view this with another microscope not shown in the figure. The resulting magnification was such that a movement of one division on the eyepiece scale corresponded to an actual deflection of the fiber amounting to 1/600 mm.

Calibration curves were obtained with four different potential differences between the plates and are shown in Fig. 2. Curves *A*, *B*, *C* and *D* correspond, respectively, to potential differences between the plates of 1.5 volts, 4.6 volts, 6.0 volts and 6.4 volts, the plates being 0.5 cm apart, and the corresponding sensitivities are 138 eyepiece divisions per volt, 805 divisions per volt, 3550 divisions per volt and 19000 divisions per volt. The special features of the electroscope have shown themselves in the remarkable linearity of these curves over the whole range of the scale, particularly in the case of *A*, *B*, and *C*. At 19,000 divisions per volt the instrument would be inconvenient to use because of the time taken for the readings to be attained, but, at 3550 divisions per volt, the steady readings were attained in about three quarters of a second after applying the potential to the fiber.

A pleasing feature of the performance of the instrument was the reproducibility of the results. Thus, for example, in obtaining the data for the curves in Fig. 2 each observation was made by reading the fiber before and after applying the corresponding potential to the fiber, and a series of readings of this kind were taken with potentials increasing by steps to a maximum and then decreasing again. Even with the sensitivity of 3550 divisions per volt the agreement of the results taken with ascending and descending potentials was very good as shown in Table 1.

TABLE 1.

Pot. applied to fiber (volts)	Potential Increasing			Potential decreasing		
	Deflected Reading	Zero	Deflection	Deflected Reading	Zero	Deflection
0.217×10^{-2}	3.88	4.60	7.2	4.45	5.15	7.0
0.433	3.20	4.70	15.0	3.55	5.05	15.0
0.648	2.52	4.80	22.8	2.65	4.95	23.0
0.864	1.75	4.77	30.2	1.90	4.95	30.5
1.078	0.98	4.78	38.0	1.15	4.98	38.3
1.293	0.25	4.83	45.8	0.35	4.95	46.0

When it is remarked that the units refer to single eyepiece divisions, it will be seen that the deflections obtained in the ascending and descending sequences agreeing as they do to two or three tenths of an eyepiece division are very satisfactory. Even the zero was constant to 0.2 division in both the ascending and descending sequences, although it varied by about 0.5 division during the whole double sequence. As a

matter of fact no serious attempt was made to keep the zero constant between successive pairs of readings, and even such zero changes as are recorded may have been due in part to slight movements of the microscope.

It is of interest to compare the measured sensitivities with those calculated on the basis of the theory given above. The theoretical sensitivities are calculable from equation 11, the formula being multiplied by 6000 (the factor to reduce actual deflection in centimeters to eyepiece divisions), and divided by 300 in order to give the sensitivity in eyepiece divisions per volt, it being understood that the values of X are to be inserted in electrostatic units. The quantities r, ϵ, and indeed the capacity of the fiber which is represented by $l/(2\log_e 2b/r)$ are not known with any considerable accuracy, and, a slight error in these would make very large errors in the calculated sensitivity near the limit of stability. However, it is possible to make a test of the theory without a detailed knowledge of these quantities. Thus, referring to equation 11, it is possible to determine the quantities ϵ and

$$(\pi r^2 \rho g \epsilon^2/l) \log_e 2b/r,$$

which we shall call K, so as to make the measured and calculated sensitivities agree for the cases A and D, the process being facilitated by the fact that the square bracket in 11 is almost zero for the case D in view of the high sensitivity. In this way we find $\epsilon = 1.23$; $K = 18.8 \times 10^{-4}$.

TABLE 2.

Curve	Pot. Diff. between plates (volts)	Field between plates (volts/cm)	Eye piece divisions per volt		
			Legitimate sensitivity (calculated)	Actual sensitivity (calculated)	Measured sensitivity
A	1.5	3	131	138	138
B	4.6	9.2	402	797	805
C	6.0	12.0	524	3430	3550
D	6.42	12.8	559	19000	19000

It is then possible to calculate the sensitivities corresponding to the cases B and C; and, in Table 2 the results are collected and the condition under which the instrument operates is further illustrated by the values of the legitimate sensitivity which may be calculated from the actual sensitivities by means of (12).

Although the quantities ϵ and K have been determined so as to make the cases A and D fit the measurements, the range of sensitivity covered by these cases A and D is so great and the sensitivity varies with X in a sufficiently complicated way to make the agreement with the cases B and C of very satisfactory significance. Further, the value $\epsilon = 1.23$ tells us that the effective point at which the electric charge on the fiber should be located for the purposes of the calculation is a point $1/1.23$ from the free end, a result well in harmony with reason when one observes that on account of the concentration of charge density on the more pointed regions in preference to other places, the effective position of the electric charge should be below the center of the fiber. The value of r can be calculated from

$$K = \frac{\pi r^3 \rho g \epsilon^2}{l} \log_e \frac{2b}{r} = 18.86 \times 10^{-4} \tag{14}$$

taking $\rho = 2.2$ for quartz, $2b = 0.5$ cm, $l = 2.2$ cm, and $\epsilon = 1.23$, we find $r = 2.3 \times 10^{-4}$. While the diameter of the fiber was not measured with any great accuracy before it was inserted, it was known to be about 1 micron. The above value of r is consequently rather large, although of the right order of magnitude. However, no great weight must be placed on agreement of the actual value of r with that calculated as above, for several reasons. In the first place, if the coating of platinum was appreciable the effective density would be greater than for quartz which would increase the value of r as calculated from (14). Then the expression $l/(2\log_e 2b/r)$ for the capacity of the fiber is only a rough approximation. Finally, the resistance of the fiber to bending acts to increase the restoring couple. Its neglect does not alter the form of the relation between S and X, but it does render our expression for K as a function of r, ρ and l incorrect, and the effect is to cause us to calculate from the value of K deduced by experiment a value of r which is too large.

A MODIFIED DESIGN OF INSTRUMENT

The characteristic features of the instrument appear to render it very convenient for use in connection with the ionization chamber of an x-ray spectrometer. Moreover, its small size renders it capable of being mounted on the arm of the spectrometer. In order to increase the ease of manipulation, particularly in relation to the mounting of the fiber, and in order to provide for a more permanent construction the modified design represented in Fig. 3 has been adopted. The weight

of the instrument has been thereby increased to some extent, but this is compensated for by the fact that the key which must necessarily be

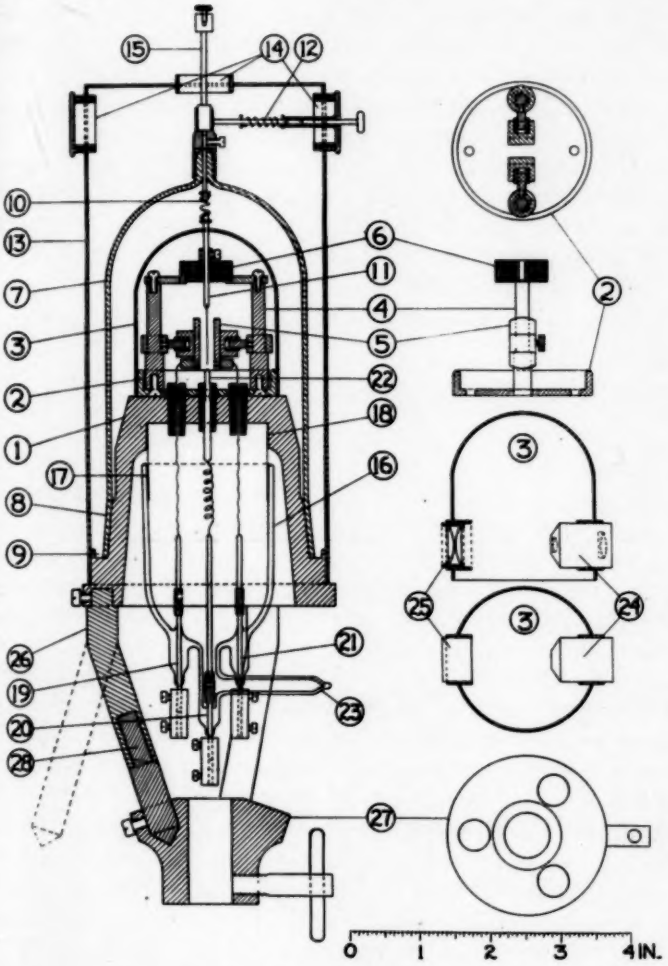

FIG. 3. *Instrument of more elaborate design.*

associated with the electroscope is incorporated as part of the construction.

The foundation of the instrument is a brass cone 1. A thin spun brass shield 3 fits over a part 2 which is screwed to the top of the cone and forms the support for the pillars 4 which carry the insulated plates 5 and the insulated fiber support 6, all being contained within 3. A pyrex glass cover 7 fits over the system and rests on a ground surface of the cone at 8, where a seal is made by means of a cement (a mixture of resin and beeswax) placed in the trough 9. A tungsten rod is sealed into the top of the pyrex cover, and carries a spring 10 by which contact is made with the lug 11 which carries the fiber. To accommodate key 12, by means of which the fiber may be connected at will to the brass cone, and to provide for a shield for this key and all parts of the system which emerge through the hole in the top of 3, an outer brass case 13 is provided. By removing the cap over the opening 14, the part 15 may be inserted at will for the purpose of making connection to external apparatus.

16 is a pyrex bell with a copper to glass seal at 17, the copper being soldered into the cone 1 at 18. In this glass bell are three tungsten seals 19, 20, and 21. Of these, 19 and 21 serve to make connection to the plates 5, while 20 connects to the brass rod 22, and enables it to be raised to a high potential for the purpose of controlling the fiber during constructional manipulation. The outlet 23 is for evacuation. The shield 3 carries two tubes, one of which carries the microscope objective 24 and the other the optical condenser 25. Both of these of course fall within the pyrex cover 7; but, by turning this cover around it was possible to find places which produced no appreciable distortion of the image produced by 24 as viewed by another microscope placed outside. The brass case 13 is of course provided with two windows in line with the optical system. It will be observed that the whole of the electroscope system is enclosed in a vacuum tight enclosure, with only one wax seal, that at 9.

The cone 1 is supported by three legs 26 on the brass piece 27, which arrangement is convenient when one desires to mount the electroscope on a stand. If it is desired to allow the electroscope to stand on the table, the three legs may be removed from 27 and turned around into the dotted position, in which they form a tripod. Suitable leveling may be brought about by the construction indicated at 28.

I take pleasure in acknowledging the services rendered by my research assistant, Mr. C. A. Kotterman, during the construction of the preliminary apparatus at the University of Chicago, and those of Mr. D. A. Bloodgood in the construction of the final apparatus at

Yale. I am also indebted to Mr. G. Bloom for his assistance in some of the preliminary experiments with regard to the optical system.[4]

SLOANE LABORATORY,
YALE UNIVERSITY, NEW HAVEN, CONN.
APRIL 12, 1925.

[4] Since this paper was written I have received, through the courtesy of Professor Siegbahn a publication "Undersökningar rörande Elektrometrar" by Gustaf Ising, published by Gleerupska Universitetsbokhandeln, Lund, in which an account is given of a single fiber electroscope of design somewhat different from that here described. The author also appears to go into the general theory of this and many other instruments. The publication had escaped me as it is not in one of the regular journals, but is printed separately as a Doctor's thesis, and is in Swedish.

OPTICAL SOCIETY OF AMERICA TREASURER'S REPORT

FOR THE PERIOD FROM JANUARY 1, 1924 TO DECEMBER 31, 1924

RECEIPTS

Cash on Hand, January 1, 1924	$ 7,618.14
Dues for 1921	5.00
Dues for 1922	45.00
Dues for 1923	290.00
Dues for 1924	1,715.00
Life Membership Dues	50.00
Donations	3,000.00
Interest on Bank Balances	233.76
Journal Subscriptions from Subscribers on Books Prior to Feb. 15, 1922	281.24
Advertising	421.00
Sale of Journals—Back Numbers	254.91
Sale of Colorimetry Reports	13.00
	$13,927.05

DISBURSEMENTS

Printing and Stationery	$ 109.82
Postage and Incidentals	27.17
Association of Scientific Apparatus Makers of United States Contract	5,551.88
Journal Subscriptions Since February 15, 1922	135.00
Cash on Hand, December 31, 1924	8,103.18
	$13,927.05

I certify that the above report is correct to the best of my knowledge and belief.

(*Signed*) Adolph Lomb, Treasurer.

Adolph Lomb, Treasurer,
Optical Society of America,
Rochester, New York.

Dear Sir:

In accordance with your request, we have audited the books of the Optical Society of America for the twelve months ended December 31, 1924.

We hereby certify that the above report of Cash Receipts and Disbursements is a true statement for the period from January 1, 1924 to December 31, 1924.

ROCHESTER, NEW YORK.
February 23, 1925.

Respectfully submitted,
(*Signed*) Naramore, Niles & Co., Auditors.

FINANCIAL REPORT OF THE JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND REVIEW OF SCIENTIFIC INSTRUMENTS

FOR THE PERIOD FROM JANUARY 1, 1924 TO DECEMBER 31, 1924

RECEIPTS

Cash on Hand, January 1, 1924		$ 4,714.79
To apply on Contract of the Optical Society		
Subscribers on books prior to February 15, 1922	$ 281.24	
Sale of Back Numbers of Volumes I-V	254.91	
Advertising collected for Optical Society	421.00	
Payments from Treasurer of Optical Society	4,724.46	
Sale of Reports of Committee on Colorimetry	18.05	
Total		5,699.66
To apply on Contract of the Association of Scientific Apparatus Makers of the United States of America		
Advertising		8,571.25
Miscellaneous Receipts		
Subscriptions since February 15, 1922	$ 1,113.54	
Sale of Back Numbers of Volumes 6, 7, 8 and 9	211.68	
On account of Helmholtz Translation (contra)	223.35	
Sundry items	8.63	
Total		1,557.20
Grand Total		$20,542.90

DISBURSEMENTS

Printer	$13,124.97	
Salaries	4,214.25	
Supplies	155.38	
On account of Helmholtz Translation (contra)	223.35	
Miscellaneous (Postage, advertising)	543.33	
		$18,261.28
Cash on Hand, December 31, 1924		2,281.62
		$20,542.90

The above is a correct statement of accounts to the best of my knowledge and belief.

(*Signed*) F. K. Richtmyer, Business Manager.

We, the undersigned, duly appointed to audit the accounts of the Journal of the Optical Society of America and Review of Scientific Instruments as at the close of business, December 31, 1924, certify that we have examined the statement of receipts and disbursements, the vouchers, check book and bank book and find them to agree with each other. We, therefore, certify that the above statement is a true statement of the accounts of the Journal of the Optical Society of America and Review of Scientific Instruments as of December 31, 1924.

January 17, 1925.

(*Signed*) Charles C. Bidwell
Walter H. Stainton
Auditors.

AN AUTOMATIC RECORDER FOR MEASURING SIZE-FREQUENCY DISTRIBUTION OF GRAINS*

By Raymond H. Lambert and E. P. Wightman

A "Sedimentation Tube" for the determination of size-frequency distribution of particles in suspension by measuring change of density was first used by Ostwald and Hahn.[1] It was greatly improved by W. J. Kelley[2] who bent the arm of the small tube at a point nearly level with the surface of the suspension into a nearly perpendicular position to that of the large tube containing the suspended material. By taking the product of the change in length of the column of liquid in the side arm and the sine of the angle made with the gravitational normal, the change in vertical height is given, which in turn is proportional to the change of mass of material settled out at any given time.

E. O. Kraemer[3] by attaching the side arm a short distance below the surface of the liquid, was able to measure the distribution of particles lighter than the medium within which they were suspended. Neither he nor Kelley have taken into account certain errors in the measurement. We shall consider these later.

In all previous methods, a scale on the side arm was necessary to read the change in length for a given time in any experiment. For a comparatively short interval of sedimentation this method is satisfactory since by use of a cathetometer the error in reading is much below other experimental errors. The error due to surface tension between glass and liquid in the small column is serious but much of it can be eliminated by not using too small a capillary, and, when water is the liquid, by first cleansing the glass with a hot solution of tri-sodium phosphate and borax, then washing clean with distilled water.

In many cases, sedimentation continues over several days and then an automatic recorder is quite necessary. Such a recording device is shown diagramatically in Fig. 1 and photographically in Fig. 3. A camera with three cylinders, F, F', F'', carries sensitized paper. Un-

* Communication No. 244 for the Research Laboratory of the Eastman Kodak Company

[1] Koll. Zeit., *32*, p. 60; 1923.

[2] Ind. Eng. Chem., *16*, p. 928-30; 1924.

[3] J. Am. Chem. Soc., pp. 2709-2718; 1924.

rolling from F' which is prevented from rotating too freely by a tension spring D, the paper passes over the idler, F'', and is wound onto F. F is rotated at a convenient rate through a series of gears B, B', by means of the motor, A. A slit, H, in the top of the camera is directly over the idler and is of such a width that sufficient time is given for an exposed field to pass by before an ensuing exposure is given. Both camera and gears are placed on a platform, I, which must be tilted in order that the paper immediately below the slit can be made parallel to

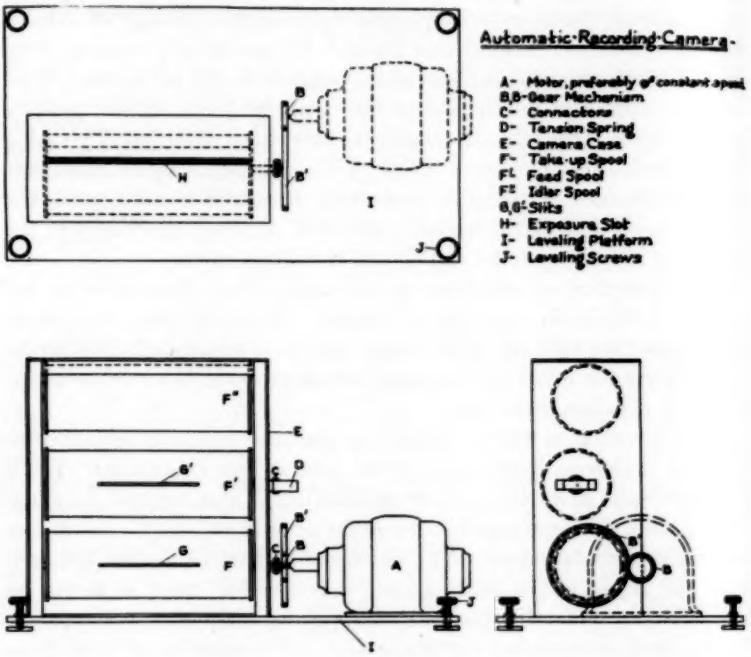

FIG. 1

the small side arm. Levelling screws, J, are used to adjust the level of the instrument. Slits, G, G', are cut in the drums to insert the ends of the sensitized paper which should be provided with suitable leaders.

In the particular instrument used, the exposure slit was 2.38 mm wide and 30.48 cm long. The paper was then regulated to move at the rate of either 2.38 mm per minute or per 10 minutes, since those were chosen as the most suitable intervals for exposure. A switch connected to the gears enables one to change the speed of the moving paper. The

exposure time for each individual record was two seconds and by using a suitable ground-glass aperture in place of the filter in a safe lamp containing a 10 watt lamp, it was found that commercial bromide emulsion on a single weight paper produced the most satisfactory results. This paper can be safely handled in a Wratten Series O safe light.

The flashing system is shown in Fig. 2. Two circuits are necessary in the system. The exposure lamp, *I*, operated on a 110 volt line, is in

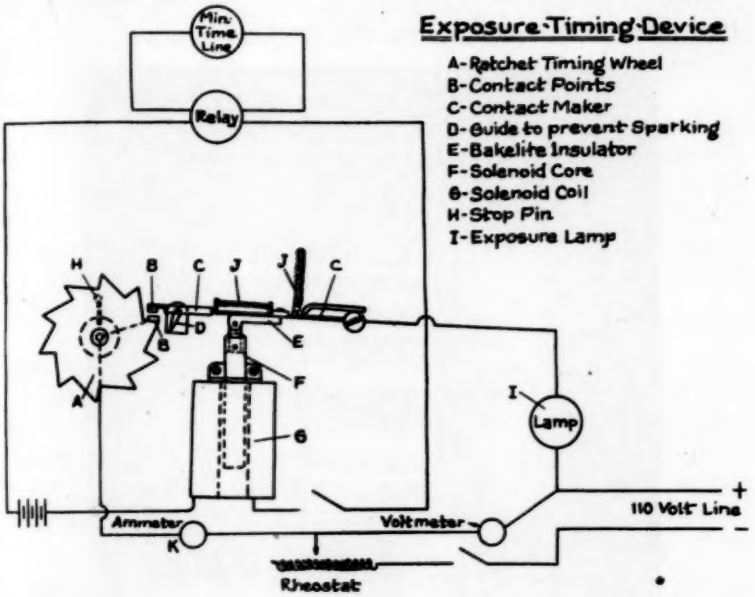

FIG. 2

series with an ammeter, *K*, cogwheel, *A*, and contact maker, *C*. A voltmeter and rheostat have also been added to keep the light source constant. The other circuit is connected by relay to a minute time line.[4] A solenoid, *G*, pulls down the bar, *F*, when the electric impulse occurs at the start of each minute. Contact is then made at *B*, *B'*, allowing current to pass through the lamp. Of the ten cogs on the wheel, *A*, *B* is the only one containing a metal contact point and is therefore the only one which

[4] Instead of a system actuated by an outside line the motor which drives the camera or an independent synchronous motor may be made, by means of certain connections through a gear system, to make and break the circuit through the lamp.

will cause the light to flash. A spring, J, is used to pull the contact maker up after the solenoid circuit is broken and another J' pushes the upper contact, B', into position ready for the next flash.

Some trouble resulted from sparking at the tips of B, B', when C was pulled back into position, especially when a high wattage lamp was used. This was quite well eliminated by a wedge-shaped piece of metal, D. A small peg, not seen in the diagram, back of C, passes down at the left of D but on rising passes up at the right. This causes the contact maker, B', to move away from B on the rising motion of C.

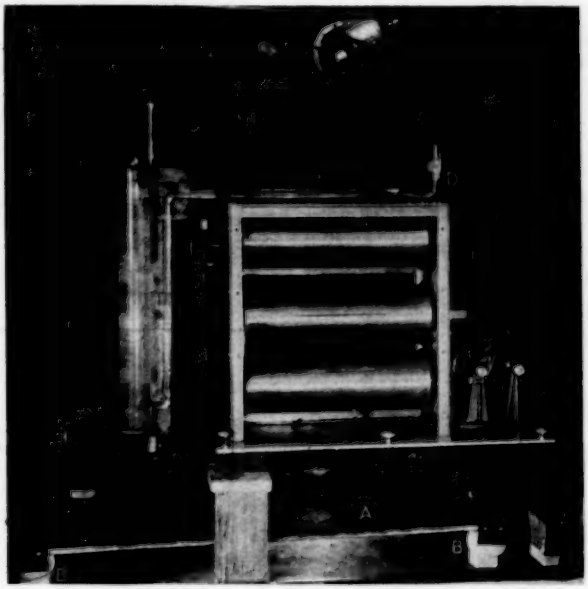

FIG. 3. *Automatic recording and sedimentation tube.*

A lock pin, H, holds the wheel rigid when minute flashes are wanted. On removing the pin, the wheel is turned by the distance of one cog each minute and ten minute intervals elapse between flashes. By using other wheels with a suitable number of cogs nearly any interval of flashing from one minute to an hour can be obtained. The time of flashing was two seconds.

Fig. 3 shows a photograph of the sedimentation apparatus and recording camera. The sedimentation tube, proper, is 44 cm high and 4.96 cm in diameter. The effective height for water suspensions is approximately

34.7 cm. The exact height, of course, depends on the density of the suspensions. The length of capillary tube for measuring the change of length of liquid column during an experiment is of necessity limited by the width of sensitized paper available. In our experiment, paper 30 cm wide was found to be convenient, since the nearly horizontal portion of the capillary tube was about 40 cm long. The cross-sectional area of its bore was about 0.0393 cm^2 and fairly uniform throughout the length of the tube. The tube was very straight. To prevent evaporation from the large surface of the liquid which would be exposed to the air, a cap, as used by Kelley, was placed in the mouth of the sedimentation tube during an experiment.

Light on passing through the side arm in the region occupied by liquid is brought to a focus on the paper, i.e., the tube containing liquid acts as a cylindrical lens. The light, however, is nearly all dispersed by the empty portion of the tube. This calls for suitable focusing before an actual experiment is attempted. With the instrument used, the distance from the bottom of the side arm to the paper was about 6 cm, and the distance of the lamp from the paper about 46 cm.

The sedimentation tube and recording camera are enclosed in a light tight air thermostat regulated to within 0.1° of the desired temperature. The temperature at which we usually work is 25°. A fan, *C*, Fig. 3, in the thermostat keeps the air well circulated. The sedimentation tube is placed on a special platform, *A*, Fig. 3, designated to eliminate vibration. This consists of an iron plate supported at the corners on soft, solid rubber, tennis balls, *B*., Fig. 3.

OPERATION OF THE SEDIMENTATION RECORDER

Before an experiment is started a calibration is first made, and one may also be taken after the experiment to insure that a record of any motion of the capillary side-arm during the experiment is obtained. The large tube is filled to the point where the meniscus of the side arm is farthest from the large tube but where it will still give an image on the paper. A flash is then taken after the water in both arms has entirely found its level. A definite amount of water is now withdrawn from the larger tube and flashes at one minute intervals are made until the water has again found a new level, when another portion of water is removed. This is continued, until the image of the meniscus moving across the sensitized paper has entirely left the paper. *A* and *A'*, Fig. 4, represent photographic reproduction of calibrations before and after an actual sedimentation. The method of using the calibration will be considered presently.

The water in the large tube is now replaced by the suspension, the meniscus first being drawn to its original position and held there by closing the stopcock, *D*, Fig. 3, at the end of the side arm. By quick operation, the recording can be started within a minute after inserting the suspension. If the material settles fast the minute flash is used and if slow it is converted to the ten-minute interval. Since the first portion of material usually settles more rapidly than the remainder, it is an advantage, even for a suspension requiring several days to settle, to run for a time on the minute system and eventually to transfer to the long time interval. This can easily be accomplished as described above and shown in *B*, *C*, *D*, *E*, Fig. 4, the photographic sedimentation record.

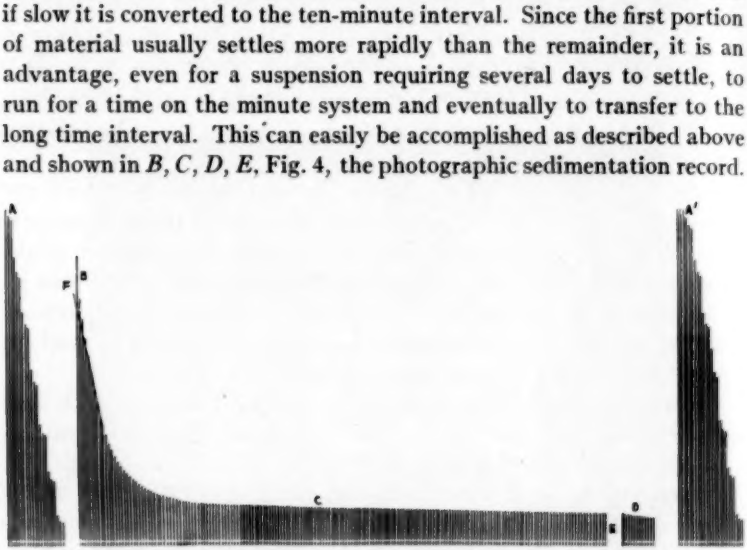

FIG. 4. *Photographic sedimentation record and calibration.*

At *B*, Fig. 4, is the initial record of sedimentation and the meniscus evidently moves quite rapidly until the portion *C*, is reached. After a series of flashes at one minute intervals, the stop pin, *H*, Fig. 2, is removed without changing the gears on the camera. After several minutes the gears are shifted and the portion of the record under *D*, Fig. 4, represents the slow speed, i.e., flashes 10 minutes apart. A small thread or wire under the capillary gave a reference line at the edge of the paper to which the meniscus was approaching.

Several errors occur in the sedimentation curve as shown in Fig. 4, for which corrections should be made before converting the data by Oden's method into a weight or size frequency distribution.

In the first place, the light source being practically at a point not far from the instrument and the capillary not being in contact with the sensitive paper, this causes a parallax in the record at points on either side of where the light falls perpendicularly. This, however, is elimi-

nated by making use of the calibration which has the same parallax error.

In the second place, as the meniscus in the capillary moves in response to the decrease of density of the suspension in the large tube, a small amount of water flows out of the capillary into the large tube. Kramer's formula for making this correction is necessary whenever the above mentioned calibration is not used but the correction eliminates itself in the equations given below. This flow, however, may also affect the rate of settling of the particles especially for experiments of short duration. This necessitates a side arm of as small a bore as possible but not so small as to magnify extremely the third principal error.

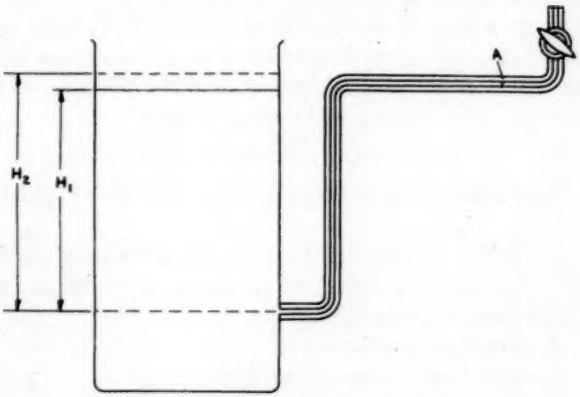

FIG. 5. *Diagrammatic sedimentation tube.*

This third error is due to a lag in the flow of liquid in the capillary because of its viscosity, which lag is greater the faster the rate of flow. From A, A', Fig. 4 it is possible to measure this lag at B, the meniscus has not travelled as far as it should have, had time enough been given to allow equilibrium to occur. This can be corrected for in the following way:

Let Δl_e be the length the meniscus should have travelled during the minute, and Δl be the length actually travelled. This will apply to the sedimentation values as well as to the calibrations. By plotting Δl_e against Δl for a number of flashes after removing a definite amount of liquid from the large tube, a straight line passing through the origin is obtained, i.e., $\Delta l_e = u\Delta l$ for any small period of time $dl_e = udl$ where u depends on the total length of the side arm. By obtaining u at successive lengths of side arm and starting with $l = o$ at the reference line on the

sensitized paper, it is then possible to convert all recorded lengths into corrected values which would be obtained if the meniscus had travelled infinitely slowly, i.e., under equilibrium conditions.

Instead of using the above equation and calculating the corrected values, an approximate method may be used of drawing a tangent to the observed curve through the known point[5] at which the sedimentation begins, and measuring the lengths where this tangent cuts the ordinates of the observed curve. This is shown in Fig. 4. This method should be theoretically correct if there is not serious parallax error, since during the early part of the sedimentation, before all particles of the largest class size have passed the level at which the side arm is attached to the large tube, the rate of sedimentation is uniform.

For a given weight of particles in suspension there is an equivalent weight of the suspending medium which, if the particles were eliminated, would be necessary to bring the meniscus in the side arm to the same level. The following relation will be seen to hold,

$$h_1 (x_1 d_1 + x_2 d_2) = [h_1 + (h_1 - h_2)]\, d_2 \qquad (1)$$

where h_1 = height of suspending medium containing particles (see Fig. 5)

h_2 = height of suspending medium in absence of particles but necessary to bring the meniscus, A, to the same level.

d_1 = density of particles.

d_2 = density of medium

x_1 = fractional volume of particles

x_2 = fractional volume of medium

and $$x_1 + x_2 = 1$$

By substitution of $(1-x_1)$ for x_2 in (1), the following relation may be derived:

$$h_1 x_1 (d_1 - d_2) = d_2 (h_2 - h_1) \qquad (2)$$

multiplying both sides by the cross-sectional area of large tube, A, we have

$$A\ h_1 x_1 (d_1 - d_2) = A\ d_2 (h_2 - h_1) = d_2 V_2 \qquad (3)$$

in which V_2 is the volume of liquid between h_1 and h_2 and hence $d_2 V_2$ is the mass of this liquid, m_2. $A h_1$ is obviously the volume of the suspension, which may be designated V_1. But,

[5] This point is obtained by adding to the final length of column after complete sedimentation the length corresponding to total weight of sediment.

$\frac{v_1}{V_1}=x_1$ where v_1 is the volume of particles in suspension. By substitution and elimination in (3),

$$v_1(d_1-d_2)=m_2 \tag{4}$$

Since $v_1=\frac{m_1}{d_1}$ where m_1 is the mass of particles in suspension at the start,

$$\therefore\ m_1=\left(\frac{d_1}{d_1-d_2}\right)m_2 \tag{5}$$

When sediment begins to fall out,

$$m_1-\Delta m_1=\left(\frac{d_1}{d_1-d_2}\right)(m_2-\Delta m_2) \tag{6}$$

Hence from equation [5],

$$\Delta m_1=\left(\frac{d_1}{d_1-d_2}\right)\Delta m_2 \tag{7}$$

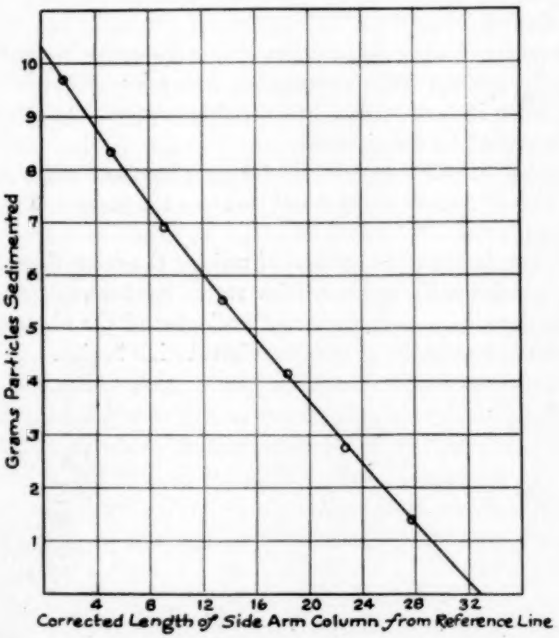

FIG. 6. *Conversion of length of arm to mass of particles sedimented.*

Referring to Fig. 4, the calibrations, A, A', are in terms of Δm_2. By equation [7], the data are converted into a mass of particles which have sedimented. A plot, Fig. 6, can be made of length of column of side arm as abscissa against the mass of particles which have settled as ordinate. From the corrected length of column, it is possible to read off on the graph the mass of particles sedimented in a given time.

The conversion of these data into size frequency distribution is then identical with Oden's mathematical treatment or with the simpler method of tangents as used by Svedberg.

It should be noted that the points plotted in Fig. 6 do not fall on a straight line. This is due to the parallax error mentioned above.

We wish to express our thanks to Dr. S. E. Sheppard for helpful suggestions in regard to this work and to Dr. L. Silberstein for advice regarding the calculations.

SUMMARY

1. A simple automatic photographic method for determining weight or size-frequency distribution of particles in a sedimenting liquid has been developed.

2. A method of correcting for an important error in the Ostwald-Hahn, Kelly sedimentation process has been worked out. This error is due to a lag in flow of liquid in the side arm of the sedimentation apparatus caused by its viscosity.

3. A method of calibrating the instrument has been emplcyed which eliminates other sources of error and shortens the process of conversion of the data.

4. An approximate rapid method of making the corrections for lag in the initial portion of the sedimentation record has been suggested. This consists in drawing a tangent to the ordinates of the observed curve through the known point at which sedimentation begins.

EASTMAN KODAK COMPANY,
ROCHESTER, N. Y.
MAY 25, 1925.

A DIRECT READING SPECTROPHOTOMETER

By Carl W. Keuffel

INTRODUCTION

It is scarcely necessary to point out the need of having available instruments with which to measure and specify color. There are two fundamentally different principles which can be used as a basis for a color measuring instrument. We can either (a) measure the physical characteristics of the stimulus which evokes the sensation of color, or (b) we can use a psychological method and match the color sensation produced by varying a known stimulus. The Spectrophotometer falls in class "a"; it measures the spectral distribution of the stimulus and is therefore independent of any psychological factors such as the sensitivity of the observer's eye, the color characteristics of the illuminant, etc. Instruments coming under class "b" are called Colorimeters. They serve a definite purpose and under the proper conditions will give useful results. Since, however, the sensation of color experienced by an observer depends not only on the stimulus but upon the individuality of the observer and upon many conditions under which the stimulus is observed, the Colorimeter cannot be considered as a primary instrument for color measurement and specification.

It has been pointed out by many authorities that the Spectrophotometer must be considered as the fundamental instrument for color measurement. I quote from H. E. Ives who in a discussion of methods of colorimetry says: "However, it is very questionable whether either of the three element (Colorimetry) methods of analysis will ever be satisfactory, beautifully simple though they are in theory. A spectrophotometric table, derived from at least 25 points, gives the only unique description of color. and it appears probable to the writer that the requirements of precision technical color measurement are most likely to be met by the development of simple and rapid means of plotting and recording accurate spectrum plots of reflection or transmission characteristics." Also, quoting from I. G. Priest: "Of whatever value the so-called 'Colorimeter' may be in special cases, it must be admitted that the fundamental basis of color specification is spectrophotometry." While it has long been recognized that the spectrophotometer is a

requisite in color measurement, providing, as Priest says, the fundamental basis of color specification, the only spectrophotometers hitherto available have been complicated instruments, elaborate in design, expensive in original cost, difficult to set up, and not calibrated to read directly.

The instrument which was developed to fill the need for a practical, direct reading spectrophotometer is called K & E Color Analyzer, and it consists essentially of:

A. The Constant Deviation Spectrometer with its wave length scale.

B. The Direct Reading Rotating Disc Photometer with its photometer scale.

C. The Light Source with its holder for the samples when reflection measurements are made.

D. The stand with adjustable holder for transparent substances.

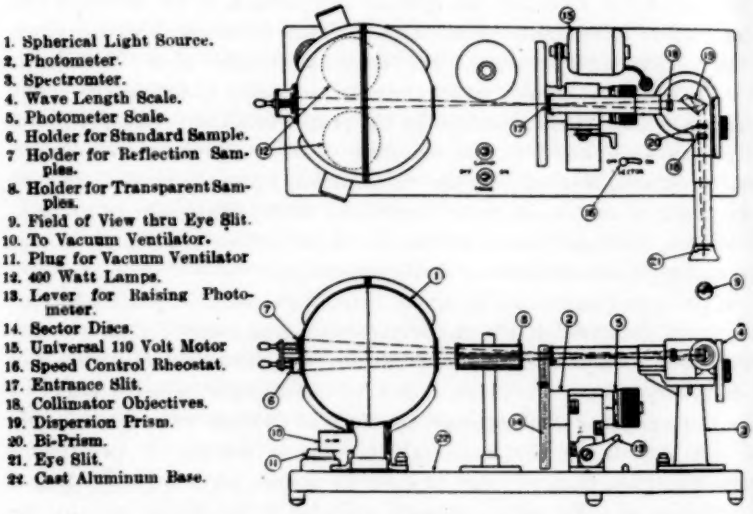

FIG. 1. *Diagram of K & E Color Analyzer.*

These parts are all rigidly mounted on a cast aluminum base plate which insures permanent alignment. A sodium flame spirit lamp is furnished for setting the wave length scale. To make the equipment complete and ready for any kind of color measurement, containers are also furnished for liquids and powders.

DESCRIPTION OF INSTRUMENT

A. The Constant Deviation Spectrometer has an adjustable entrance slit (17) and a fixed width exit or eyeslit (21). For reflection measurements the slit widths are comparatively wide, purity of spectrum being sacrificed for illumination because few spectral reflection curves have sharp dips. The optical elements consist of the two achromatic collimator objectives (18), the dispersion prism of dense flint glass (19) and the bi-prism (20) which produces the divided field.

The dispersion prism is mounted on a rotating platform having a conical center. This platform is actuated by the drum (4) carrying the wave length scale ranging from 420 to 700 mμ, graduated every 5 mμ. Blue and Red filters are provided, either of which can be inserted in the beam at the eyeslit to cut out stray light when measurements are being made at the ends of the spectrum. All parts are constructed rigidly and fitted accurately. The only adjustments necessary are on the entrance slit, the bi-prism and the wave length scale drum. These adjustments when once made are secured rigidly so that under ordinary conditions the instrument keeps its adjustment for a long time.

B. The Direct Reading Rotating Disc Photometer consists of two sectored discs, with two opposite openings, which revolve around the same axis and in the same direction. The effective angular opening between discs is set while they are in motion by means of turning the knurled head to which the photometer scale is attached.

One sector (shown by dotted lines Fig. 2) is larger in diameter than the other (solid lines) and cuts the upper beam a constant amount while the amount of the lower beam passed can be varied from 50 per cent to zero. (The two beams are represented by the open circles, Fig. 2). The constant amount cut off by the larger sector is such that the relative change between the upper and lower beams can be varied between 0 and 110 per cent. The photometer scale reads from 0 to 110.

The fact that the photometer reads above 100 to 110 per cent is of decided advantage in measuring a sample having very high reflection or transmission values, as the matching point can still be approached from both sides and the setting made as accurately as with the lower values. It also simplifies the calibration of the instrument as 100 per cent can be used as a reference point.

When measuring dark samples the center line of the sectors can be lowered by means of a lever (13, Fig. 1). (Now the beams are represented by the cross hatched circles). This doubles the illumination and increases the accuracy of the settings as the sectors are so made that the scale will then read four times the actual value.

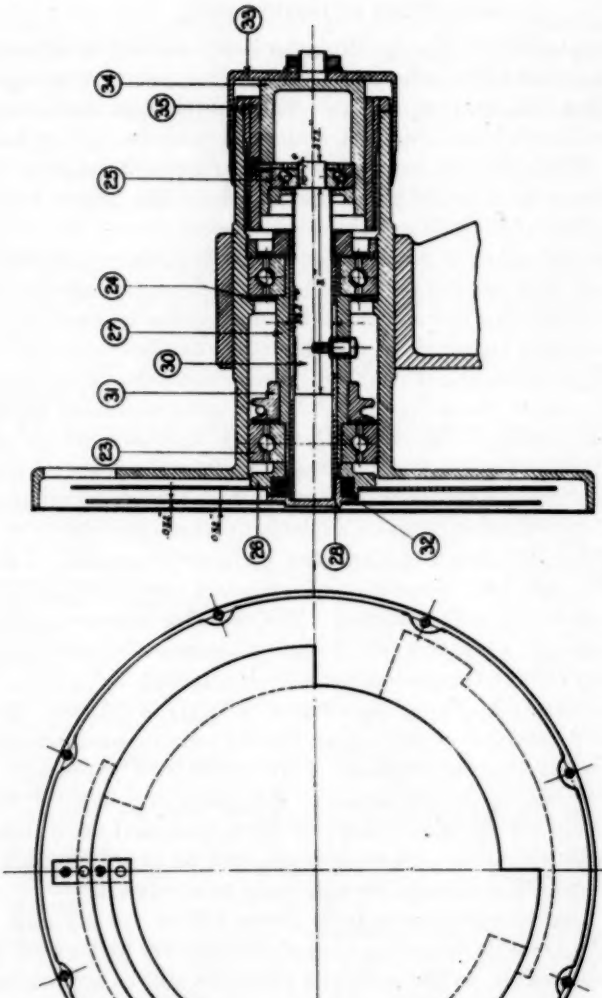

FIG. 3. *Sectional view of photometer showing mechanical details of construction.*

FIG. 2. *View of photometer with cover removed showing sectored discs.*

The discs are rotated at high speed through an endless rubber belt by a motor which runs on either 110 AC or 110 DC, and the speed is controlled by a rheostat. All fast rotating parts run in ball bearings (23) (24) (Fig. 3) and end thrust is also taken up by a ball bearing (25).

The mechanical details of the sector photometer are shown in Fig. 3. The larger sector is attached to the sleeve (26) (27), the smaller sector is attached to an inner tubular sleeve (28). The plunger (30) carries a pin which engages in a straight, lengthwise slot in the inner sleeve and in a cam slot cut into the outer sleeve. The outer sleeve, inner sleeve and plunger are rotated by means of pulley (31). A helical spring at (32) takes out the backlash between the pin and the two slots. The angular relation between the two sectors is changed by moving the plunger in translation along the axis of rotation. This is done by turning the knurled ring (33) to which is attached a sleeve (34). As the ring is turned, the sleeve advances, in the cam (35) and through the ball thrust bearing (25), translates the plunger along the axis. The accuracy with which the Photometer performs is dependent upon the excellence of the design and workmanship of the mechanical parts, and extreme care is taken to build the instrument to withstand long use without appreciable wear.

C. The light source consists of a spherical housing (Fig. 1) of cast aluminum well ribbed on the outside to radiate the heat generated by the lamps. Two 400 watt lamps (12) illuminate the inside of this sphere, which is coated with a special white paint to give bright and diffuse reflection. Special shields are provided so that no direct light from the lamps reaches the sample. This unit is ventilated and kept cool by a motor driven suction fan (attached at 10), which is furnished with the outfit. The sample holder is so constructed that a stream of cool air passes in a thin space between the sphere and the sample holder and over the sample. In this way the sample is kept cool even though it is very close to the light bulbs. The standard white (magnesium carbonate) is held in the lower holder (6) and the upper holder (7) carries the sample for reflection measurements. The unit as constructed insures approximately diffuse illumination for reflection samples. When transmission measurements are made both holders carry magnesium carbonate blocks.

D. The stand is rigid and the holder (8) for tubes and cells fits the stand so that the tubes or cells can be lined up quickly without much adjustment.

The tubes for holding liquids (Fig. 4) have a glass liner cemented into a brass sleeve. The glass end plates are plane polished and held in place with screw caps. The liquid sample comes in contact with glass only and the tubes are easily cleaned and filled. The lengths usually used are 100 mm, 30 mm and 10 mm, and they are ground to these lengths within an accuracy of 1/10 of 1 per cent.

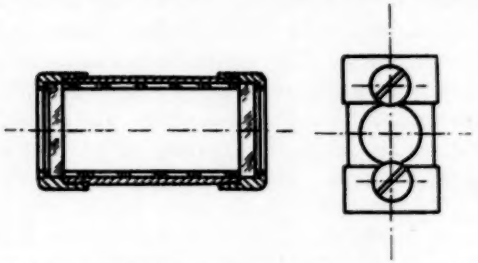

FIG. 4. *Sectional view showing construction of tubes.*

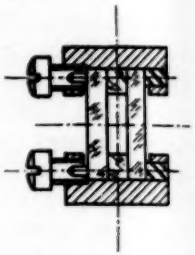

FIG. 5. *Two views showing construction of cells.*

The cells for holding liquids (Fig. 5) consist of three plane parallel glass blocks, held together with clamps. A cylindrical hole 10 mm in diameter in the middle block, forms the cavity which holds the liquid. These cells are used in the following thicknesses:

3 mm plus or minus 0.003 mm and 1 mm plus or minus 0.002 mm.

Both types of container have proven very practical in actual use.

ADJUSTMENT OF INSTRUMENT

Although the Color Analyzer is constructed rigidly and will not get out of adjustment easily, yet it is necessary and easy to check all adjustments frequently.

The adjustment of the wave length scale is checked by means of the sodium flame lamp, which gives the spectral D line. The wave length scale is marked with an extra graduation at 589 mμ to facilitate the adjustment. Any other monochromatic light source can of course be used to check the wave length scale. The cylinder carrying the wave length scale is moveable on its shaft and can be locked when properly positioned with respect to its index. This provides the means for adjusting the wave length scale.

The photometer scale must read zero when the photometer discs cut out all the light of the lower beam, i.e., when one-half of the divided field appears black. The drum carrying the photometer scale is moveable about its shaft and can be locked when properly adjusted.

The other major adjustment which must be checked is the equality of brightness of the two beams of light. To do this it is necessary to place a block of magnesium carbonate in each sample holder and see that these blocks are properly illuminated with diffuse light, i.e., the shields between the lamp filaments and samples must be in place.

When the two fields appear matched in brightness the photometer scale should read 100 per cent. If it does not read 100 per cent the adjustment is made at the entrance slit by changing the relative slit width of the upper and lower beams. The equality of brightness of the two beams must be checked throughout the spectrum and if it is found that the photometer reads high at the red end of the spectrum and low at the blue, or vice versa, the correction must be made by adjusting the bi-prism angularly about the axis of the telescope.

ACCURACY OF INSTRUMENT

The instrument gives accurate results when properly adjusted. Transmission measurements on standard glasses with various individual instruments have checked well within the limits of the precision of reading.

The absolute accuracy of reflection measurements depends to a certain extent upon the illumination of the sample as provided by the light box. Reflection measurements taken on the same samples on different Model E Color Analyzers agree very well. However, since the illumination of the samples is not perfectly diffuse, the results on glossy samples will not check with results obtained where the sample has been illuminated with perfectly diffuse light.

This factor of the method of illuminating the sample has not been properly taken care of in making spectral reflection measurements in the past, and probably accounts for the difficulty experienced by observers with different instruments who have attempted to check measurements on the same set of samples. It is difficult to devise means for illuminating a sample with perfectly diffuse light and at the same time get enough brightness with which to make spectral reflection measurements. It may be better practice to specify some other arbitrary way of illuminating the sample for spectral reflection measurements and in addition get some arbitrary measure of gloss in order to completely specify the reflecting properties of a sample.

PRECISION OF MEASUREMENT

The precision with which an average observer takes measurements can be judged from the following:

The probable error for samples reading between 10 per cent and 90 per cent for an average of five settings at any one wave length between 480 mμ and 700 mμ is about ±0.8 per cent of the reading. The probable error below 480 mμ is greater than this due to the lower brightness at this end of the spectrum and is about ±1.2 per cent of the reading.

An average of 10 readings reduces the probable error considerably. The precision of reading depends somewhat on the experience of the observer, but the results given above can be obtained with very little practice.

SUMMARY

The K & E Color Analyzer is accurately made, rigidly constructed, easy to adjust and convenient to use.

It is a complete spectrophotometer in one unit and provides a rapid and precise means of obtaining a complete color analysis of any sample, liquid, or solid, transparent or opaque.

It is being used for color standardization and factory control work in many of the industries on the following products: paints and varnishes, dyes and dyed fabrics, oils, both mineral and vegetable, soaps, sugars, honeys, etc., even artificial teeth are graded with the Color Analyzer. A few of our progressive hospitals are making use of this instrument in their research work and it is part of the equipment in the physics, chemistry, physiology and psychology laboratories in many universities.

TECHNICAL DEPARTMENT,
KEUFFEL & ESSER CO.,
HOBOKEN, N. J.

A NEW CAPILLARY ELECTROMETER, AND THE RELATION BETWEEN THE MERCURY DISPLACEMENTS AND THE RADIUS OF THE TUBE

CLARENCE E. BENNETT

ABSTRACT

The ordinary Lippmann type of capillary electrometer consists of a single fine tube in which the rise of mercury depends upon the impressed voltage for small voltages up to one volt. The new capillary electrometer here discussed is an inverted U tube device whereby direct readings of the pressure difference between the two mercury levels can be made, thus obviating the necessity for assuming that the large surface in the reservoir is unchanged. Its advantages lie in its comparative simplicity, its sensitivity (one volt corresponding to 1.5 cm deflection compared with .5 cm for an ordinary Lippmann type instrument), and its direct readings. The instrument is a result of an investigation into the laws of capillary electrometric phenomena. Lippmann's voltage-height relation is verified in an independent manner and extended to tubes of different radii. A new relationship is found and extended to tubes of different radii. A new relationship is found, that between the rise of the mercury and the radius of the tube. A simple curve represents this result, but a simple analytic expression does not exist due to the extreme complexity of the phenomenon as theory shows.

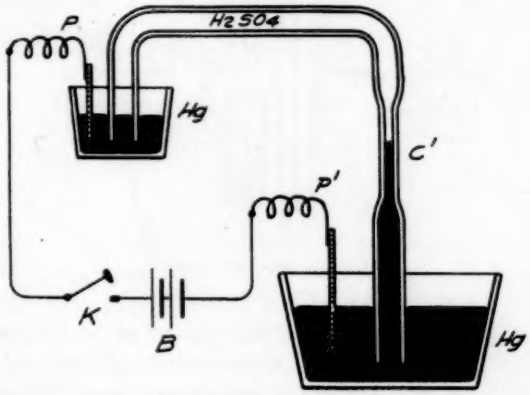

FIG. 1. *Lippmann's method.*

INTRODUCTION

Probably the best known type of capillary electrometer is Lippmann's, the elementary principle of which can be seen in Fig. 1. The two containers hold mercury and the connecting tube contains dilute sulphuric acid on top of the mercury surfaces with which electrical

contact is made by the platinum points P and P'. When the key K is closed the capillary thread C drops or rises a little depending on the direction of the current. C rises when P' is positive and falls when P' is negative. The amount of rise or fall of the mercury thread depends upon several factors, of which the most important seem to be the impressed voltage and the radius of the tube. The relation between the

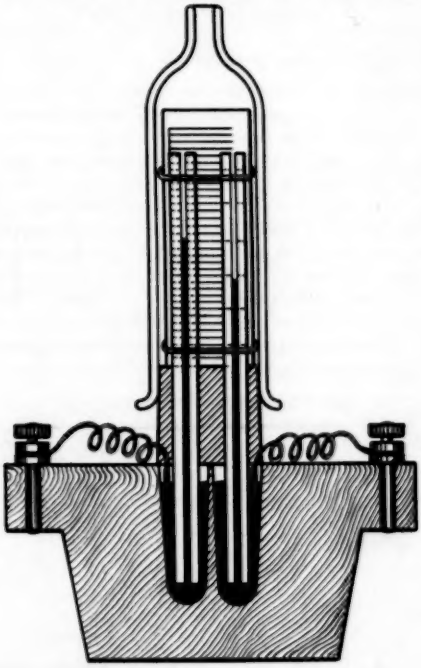

FIG. 2. *Experimental apparatus.*

change in height and the voltage was discovered by Lippmann,[1] but as far as can be ascertained the radius-height relation has not been previously determined. Lippmann also showed that the sensitiveness is independent of the concentration of the electrolyte.[1]

Experimental

A capillary tube about 20 cm long was cut in half and one end of each half was inserted into a rubber stopper. A glass mm scale was fastened behind these two tubes and the stopper was then inserted as a plug in the end of a wide mouthed drawn out glass tube, the upper end of

[1] An. de Chem. et de Phys. (5) *5*, p. 494.

which was attached to an aspirator as can be seen in Fig. 2. The projecting ends of the capillary tubes were lowered into reservoirs of mercury made of cut-off test tubes sunk into holes drilled in a wooden block. Very pure mercury is necessary and electrical contact is made by platinum points connected to binding posts. The large glass tube was filled with dilute sulphuric acid, and an aspirator connected to the top of the instrument drew two fine threads of mercury up into the capillary tubes. By this form of apparatus tubes of various radii can be readily interchanged for purposes of comparison.

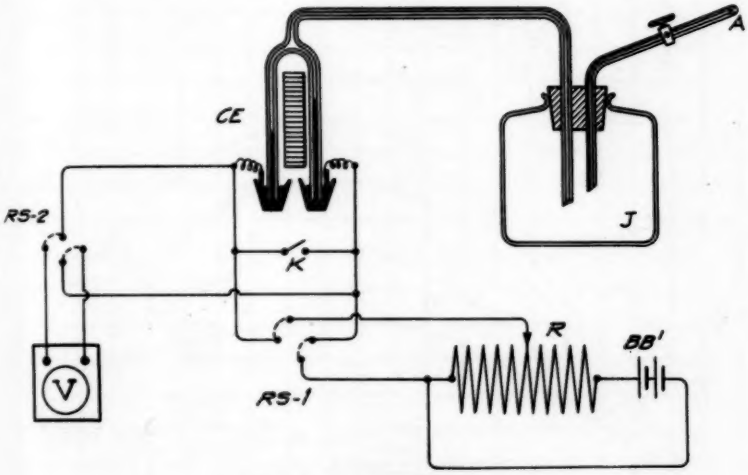

FIG. 3. *Layout of apparatus.*

The radius of the bore of each tube was determined indirectly by the method of weighing a mercury thread of measured length. Seven pairs of tubes were prepared, their radii being as follows:

Tube No. 7, radius in cm, .00540; No. 8, .00918; No. 9, .00931; No. 6, .0146; No. 5, .0216; No. 4, .0481; No. 3, .0774.

Each of these pairs was set up in the apparatus in turn, and the electrical connections were made as shown in Fig. 3. *CE* is the capillary electrometer, *B* and *B'* are dry cells in circuit with the resistance *R* used as a potential divider, and *RS*−1 is a reversing switch. The impressed potential was measured by a sensitive voltmeter *V* shunted across the electrometer through the reversing switch *RS*−2. A light behind the instrument illuminates the glass scale on which deflection readings are taken with the aid of a telescope. Before the bottle *J*

was included in the aspirator circuit, small temperature changes affected the readings to quite an appreciable extent. In fact, a rise in temperature of only a few degrees would cause the mercury threads to entirely disappear, and overnight temperature changes in the building would cause full scale deflections of seven or eight centimeters. It was found that a large volume of air in the circuit absorbed this pressure change without affecting the readings and this was very satisfactorily accomplished by the use of the bottle *J*.

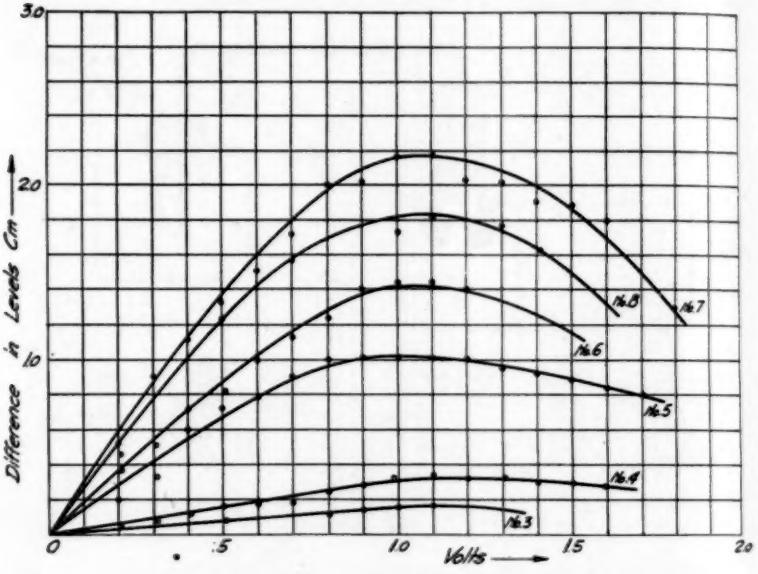

FIG. 4. *Voltage-height relation for different radii.*

In very small tubes a bothersome sticking of the mercury to the glass walls of the tube is encountered. In several instances this sticking effect was overcome by merely short circuiting the two electrodes with the key *K* between successive readings. The fact that the effect is not noticeable until the instrument has been used for some time is significant since it shows that the phenomenon is not caused simply by the viscosity of the mercury, but that the electric charge has something to do with it. It may be that the viscosity of the mercury changes upon electrification. Another characteristic of the phenomenon which seems to point toward the electric charge as the cause is that if the instrument is left standing, perhaps overnight, after this sticking occurs

the effect disappears until the instrument is used again. The smaller the tube the more marked is the effect.

Varying voltages from 0 up to 2 volts by steps of .1 volt were applied to the electrometer for each of the seven sets of tubes and the corresponding differences in level of the two mercury threads for direct and reversed current are plotted against the voltage in Fig. 4. The curves in this figure verify Lippmann's voltage-height relation in that the deflections increase up to a maximum at approximately one volt and then fall off again for further increases in voltage. It must be

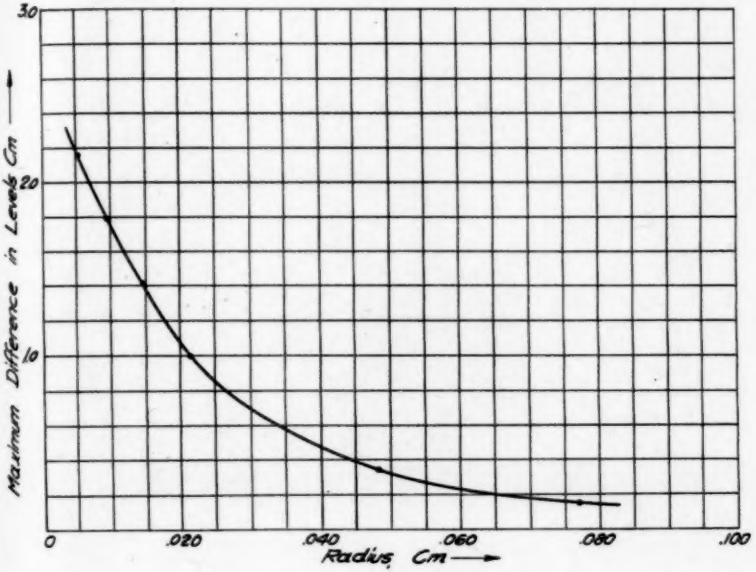

FIG. 5. *Radius-height relation.*

remembered, however, that Lippmann's deflections were the changes in level of the mercury in one tube only, whereas these readings are the differences in level in the tubes at the same time. The plotted points are averages of direct and reversed current readings. Furthermore, these results show that the maximum occurs at approximately one volt, whatever the size of the tube.

An attempt was made to see what happens beyond two volts, i.e., to see if the phenomenon repeats itself, but unfortunately this cannot be determined, for as the voltage is increased beyond 1.9 or 2.0 volts,

bubbles, probably of hydrogen, rise up apparently from within the mercury, but more likely around it just inside the glass wall, and affect the surface tension of the liquids. After this happens the readings cannot be continued for the instrument becomes "dead" and voltage changes have no effect whatever on the mercury threads. If the mercury is lowered into the cisterns and drawn up afresh, the instrument regains its sensitiveness.

The maximum ordinates of the curves in Fig. 4, all of which occur at approximately one volt, are plotted against the radii of the several tubes in Fig. 5. This curve represents the radius-height relation previously referred to, and although it is simple in appearance, no simple

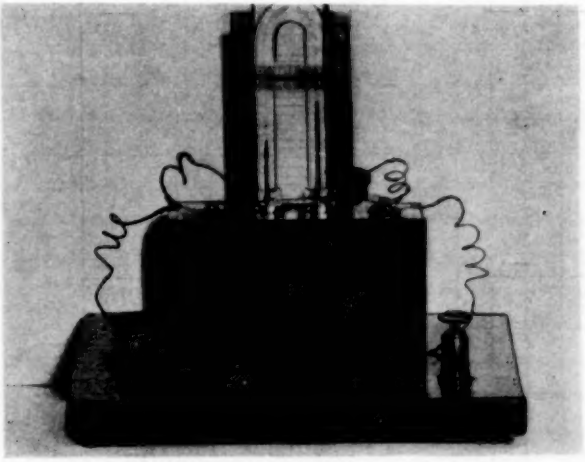

FIG. 6. *Photograph of electrometer.*

analytic expression for it has been found. Of the many attempts to fit an empirical equation to it, nothing short of a power series resulted. On the other hand, upon consideration of the number of factors which are probably involved, a simple equation would not be expected to represent the relation.

THE NEW CAPILLARY ELECTROMETER

Due to the fact that by this form of experimental apparatus direct readings were made possible and secondly, that by the use of fine tubes extreme sensitivity was obtained, it was thought worth while to construct a capillary electrometer of new form embodying these desir-

able features. Fig. 6 shows the final instrument, the construction and operation of which are self evident from the photograph. The selection of the size of tubing was made in accordance with the preceding data. A tube of about .012 cm radius of bore was used and proved very satisfactory. Smaller tubes than this exhibit the sticking effect to a very pronounced degree, and larger ones decrease the sensitivity. From .010 to .020 cm is a practical range for tube radii. The chief difficulty encountered was that of filling the tube with mercury and acid above it, at the same time excluding air bubbles. The finished instrument was compared with an Eppley capillary electrometer of the Lippmann type and proved far more sensitive, giving a deflection of about 15 mm corresponding to 5 mm for the Eppley. The instrument also compares more favorably in such features as size and manipulation while at the same time it is capable of giving very reliable results.

Due acknowledgement for the contents of this paper is made to the kind supervision and very helpful assistance of Professors Carl Barus and A. de F. Palmer, both of the Phycics Department of Brown University.

BROWN UNIVERSITY,
PROVIDENCE, R. I.

Fused Quartz for Astronomical Mirrors.—The many advantages offered by the use of clear fused quartz for astronomical mirrors were outlined by Professor Elihu Thomson, director of the Thomson Research Laboratory of the General Electric Company at Lynn, Mass., before a meeting of the Franklin Institute of Philadelphia. At the meeting he received the Franklin Medal for his achievements in the electrical field.

"For twenty-five years," said Professor Thomson, "I have borne in mind the great desirability of procuring fused silica disks instead of glass for astronomical mirrors. It is its low coefficient of expansion and its consequences which confer such great superiority as the silica disk possesses. This may be stated under several heads.

"1. Disks require but little annealing, while with the large glass this is a matter of great difficulty.

"2. They can be rough ground by a carborundum wheel without danger of fracture, an operation difficult with glass and rarely resorted to.

"3. The disks can be made very thick and rigid more easily than with glass.

"4. The fine grinding (or smoothing before polishing) is carried on with great facility, and the surface before polishing is usually of finer grain than with glass. The fused silica is considerably harder than glass, and is not so easily scratched.

"5. The polishing proceeds readily and can be carried on regardless of temperature changes. Incidentally, there is less liability of scratches forming in polishing.

"6. In very accurate work, figured by polishing, as in high grade surfaces of astronomical mirrors, the polishing and testing need not be interrupted as with glass by long rest periods, with the mirror disk kept jacketed in felt for equalization of temperature. This is very important and involves great saving of time.

"7. In service, none of the precautions against temperature variations and distortions arising therefrom are needed, and even in solar work with full sunshine on the mirrors, no evil result follows.

"When it is remembered that it took two years of testing and polishing for figure, involving long interruptions for equalization of temperature, to produce the 100-inch glass mirror mounted at Mt. Wilson, near Pasadena, California, the advantage offered by fused silica is evident.

"The optician will welcome the possibility of obtaining so-called flats of desired sizes, not subject to temperature distortion, while the making of accurate flat surfaces is evidently greatly facilitated. The silvering of surfaces of fused silica appears to be no more difficult than with glass, with the advantage, however, that the former can be warmed without risk when such warming is needed to assist the formation of the silver deposit."

Calculation of Objectives Composed of Two Cemented Glasses.—The calculation of such objectives would be rendered more rapid if one knew in advance the influence of terms higher than the third power on the spherical aberration, and those of the thicknesses of the glasses. It is shown that for the combinations of glass ordinarily used for the objectives of geodetic instruments, binoculars, and the like, this influence is quite constant and may be tabulated.

Knowing in advance the corrections due to the causes cited, one knows exactly what values to assign to the Seidel sums of the third order for spherical and for chromatic aberration expressed for a "thin" system. In this case the sums take a simple form, and use can be made, for example, of the parameters φ_1 and Q_2 employed by M. Turrière in his "Optique Industrielle" (Appendix). These special graphs, of which but two are shown, make it possible, with the aid of a suitable parabola, to obtain the value of Q_2, knowing φ_1 and the first Seidel sum for a "thin" system, with an approximation sufficiently good for practical purposes. As the influence of higher order terms, and of the thicknesses, has been taken into account it is not necessary to make any tentative adjustment to obtain definitive specifications for the system.

The calculations are reduced to a minimum, and may be made for the most part on a slide rule. An objective for a Galilean binocular, for example, takes but little more than an hour to calculate. [H. H. Slussareff, Trans. Opt. Inst. Leningrad. *4*, No. 31, 1925. (In Russian with the above abstract in French.)]

G. W. MOFFITT

AN ALL COPPER D'ARSONVAL GALVANOMETER WITH SMALL THERMAL EMF

By W. R. Coley

When using high sensitivity galvanometers in low resistance circuits, annoyance and error are caused by small resultant thermal electromotive forces within the usual galvanometer circuit.

In tracing the circuit through the galvanometer itself, from binding post to binding post, various metals are encountered; for example: brass, copper, phosphor-bronze and silver. There may be as many as ten contacts of dissimilar metals, all of them in series between the terminals of the galvanometer. Unless all parts of the galvanometer are at the same temperature, a resultant thermal emf will be set up. It is quite often difficult to avoid such temperature inequalities, though the differences may usually be kept to within perhaps a degree centigrade by properly locating the galvanometer and, if necessary, lagging it.

At the suggestion of members of the staff of the Bureau of Standards, the writer constructed a galvanometer wherein all parts comprising the electric circuit were made of copper. This procedure results in making the galvanometer nearly free from thermal emf as will be shown in the following test.

Two galvanometers of the type shown in Fig. 1 were compared, one of them supplied with the usual system and the other with a system and circuit constructed entirely of copper. In Fig. 2 are shown diagrammatically the various materials comprising the circuit of the usual galvanometer. All of these parts are of copper in the all-copper circuit instrument.

Tracing the circuit from the right hand binding post through to the left hand binding post of the galvanometer of Fig. 2, the metals passed over in order are brass to copper, to phosphor-bronze, to brass, to phosphor-bronze, to silver, to copper, to silver, to phosphor-bronze, to brass, to copper, to brass. It is apparent, from this large number of contacts of dissimilar metals, that any temperature inequalities are likely to result in thermal electromotive forces.

Before testing the two galvanometers, they were first short circuited by clamping a piece of No. 8 B. & S. copper wire across the terminals

of each instrument. By shunting their magnets, they were adjusted to be critically damped when thus short circuited. That the two galvanometers were then essentially alike as to characteristics is indicated below.

Characteristics	Usual Galvanometer	All Copper Galvanometer
Sensitivity in Megohms	235.	217
" m/m Microvolts	14.6	14.4
Period in Seconds	6.7	6.8
External Critical Damping Resis.	0.	0
Upper Suspension	Silver .0007"	Copper .0007"
Lower Helical Lead	" .0007"	" .001"
Galvanometer Resistance Ohms	16	15

In order to compare the thermal electromotive forces in the two galvanometers, small electric heaters were wound around various parts of each instrument as shown in Fig. 2. One heater, *A*, surrounded the upper torsion pin. Another heater, *B*, surrounded the connector carrying the lower helical lead, and a third heater, *C*, surrounded a connection leading to one binding post. Near each heater a small thermocouple was embedded to measure the local temperature. These thermocouples were of No. 36 B. & S. gauge and were embedded in the metal parts to a depth of 1/16 of an inch.

The galvanometers were set up at a meter distance from a lamp and scale and heating currents applied to the various heaters, either separately or in combination. The emf of each small thermocouple was measured with a potentiometer and the corresponding temperatures determined from the thermocouple calibration curves. All readings were taken after establishing thermal equilibrium. The room temperature was 25° C. The results are tabulated below.

	Thermocouple Temp. °C			m/m Deflection		Microvolts	
	A	*B*	*C*	Regular Galv.	All-Cu Galv.	Regular Galv.	All-Cu Galv.
(1)	34.2	26.8	26.8	450	4	30.8	.3
(2)	39.8	39.8	31.2	32	8	2.2	.6
(3)	29.0	31.0	30.5	356	10	24.4	.7
(4)	30.5	38.8	29.2	484	22	33.2	1.5
(5)	29.8	31.0	37.5	404	36	27.6	2.5

The temperature differences set up in the various parts of the galvanometer, in the above test, were purposely made greater than would be

ordinarily encountered in practice, in order to exaggerate the effects in a galvanometer circuit made up of various metals.

Actual working conditions have shown that deflections as large as 100 millimeters are encountered unless precautions are taken to enclose the galvanometer.

By the use of all-copper galvanometers, the deflections, obtained under conditions which produce 100 millimeters in the ordinary galvanometer, are never larger than 5 millimeters. Since there are no disadvantages in using this type of instrument, it is a very much more satisfactory galvanometer for use in all low resistance circuits.

LEEDS AND NORTHRUP COMPANY
PHILADELPHIA PA.

Ionisation et Resonance der Gaz et des Vapeurs by Leon Bloch. —Volume 11, Conferences Rapports de Documentation sur la Physique (223 pp) Le Presses Universitaires de France (Price 25 fr 50 in France).

This book covers a field considerably broader than the title would indicate. The first chapter treats briefly phenomena such as scattering of electrons and mobility of ions and their interpretation from the classical viewpoint. The following three chapters give a detailed discussion of methods and results of measurements of critical potentials in monatomic vapors together with the interpretation of the results in terms of spectrum theory. Only brief mention is made of the use of critical potential methods to determine the lower x-ray levels as this subject will be treated in detail in another volume of this series. Chapter 5 deals with the phenomena of electron currents in diatomic gases which is a very controversial subject. Recent results open to question many of the conclusions which were generally accepted when this book went to press, (May 1924). The last two chapters are on the photo-electric effect and on the equilibrium between electrons, atoms and radiation. They cover a wide range of experiments and theories in a concise manner. The need of quantitative measurements of effects produced by electron collisions, radiation, and interatomic collisions is emphasized. The book includes an extensive classified bibliography of the subjects covered.

This field of research is developing so rapidly that it is impossible to publish a general survey which is entirely up to date. Dr. Bloch has minimized the difficulty by maintaining a conservative and unbiased attitude toward conflicting viewpoints. He is to be congratulated on producing a useful and interesting volume.

F. L. MOHLER

Introduction to Theoretical Physics.—Arthur Haas Ph.D. Professor of Physics in the University of Vienna. Translated from the 3rd and 4th German editions by T. Verschoyle, M.C., B.Sc., A.R.C.S. With a foreword by Prof. F. G. Donnan, F.R.S. xiv+331 pp. E. P. Dutton & Co. New York.

This is a translation of a work which has already reached its fourth edition in the German. The author, in a special preface to the English edition, presents it as "an exposition which, while modern in treatment and outlook and not too comprehensive, would give a survey of the present state of theoretical physics as a whole, so as to lead to an exact understanding of the fundamental principles and the chief problems of the science, without going too far into details."

The first part of volume I is given to mechanics, and treats of the motion of a particle, the general principles and equations of d'Alembert, Lagrange, and Hamilton, and the motion of rigid bodies and of deformable bodies, with which are interspersed mathematical chapters on vectors and vector fields, potential theory, and the general theory of vibrations and waves.

The second part covers the fundamental laws of electricity and magnetism, the theory of the electromagnetic field, and the electromagnetic theory of light (for homogeneous media).

At the end of the book is an appendix giving a brief summary of each chapter, and also a synopsis of the uniform notation used.

The broad field covered by the book has been presented in a unified, logical, and readable manner. An important feature of the exposition is the expressed aim of the author of having the "easy intermediate steps purposely always carried out in full." This is most satisfactory, as the tendency has too often been the omission of even the difficult "intermediate steps"—so far, even, as seriously to diminish the value of many a book to the student who hopes to learn something of the subject by reading it. The additional material is not bulky, and greatly improves the clearness of presentation. The vector notation used throughout is most appropriate to the modern treatment of the subject.

But an unfortunate characteristic of a book of this wide scope, unless it assumes encyclopedic proportions, is the inevitable lack of more extended descriptive text to form the flesh and blood on the bones of the mathametical structure. For example, an admirable derivation is given of the Poinsot ellipsoid of inertia, but the usual descriptive treatment of the free motion of a rigid body in terms of the rolling of this ellipsoid on a plane has been omitted, and much of the significance of the ellipsoid is thus lost. There is some question as to the advantage gained from as copious a use of italicized words as is indulged in.

The translation of the remainder of the work will be awaited with great interest.

PIERRE MERTZ

NON-POLARIZABLE ELECTRODES FOR PHYSIOLOGICAL PURPOSES

H. C. Stevens and Enoch Karrer

In a previous paper we have described an improved muscle nerve chamber which allowed the use of a movable, non-polarizable electrode. It is these electrodes that we wish to describe in detail here. We have given some reasons in our previous paper which prompted us in the development of the muscle nerve chamber and of the electrodes. The so-called boot electrodes were unsatisfactory for our purpose because, first, of the size and form which made it difficult to use them in our chamber and to stimulate at several points close together; second, because the point on the nerve that is actually stimulated is not definite and is always very asymmetrical with respect to the nerve. We wished the electrode to be movable to any desired point on the nerve; to eliminate diffusion of the stimulating current as far as possible; to be non-polarizable and to stimulate symmetrically around the nerve over a very narrow section and at adjacent points that may be occasionally very near each other. It may be noted that these requirements are only partially fulfilled by the boot electrode and by the Lucas[1] electrode, to which we have referred in our previous paper. The details of our electrode are apparent by referring to Fig. 1. Two sheets of celluloid were cut in the form *a b c d e f g h* represented in Fig. 1. Between these sheets were interposed narrow strips of celluloid, approximately 0.5 mm thick, laid flush with the edge.

The resulting compartment had a shape shown in the view to the left, with the side *i j* sloping so that air bubbles could not easily form. It communicated through the slots with the solution in which it may be immersed (in our case Ringer's). The nerve was laid in the slot. Another compartment was built upon this of three strips and a sheet of thin celluloid of the size and shape of the rectangle *a b g h*. This latter compartment contained only the zinc electrode *mn* and zinc sulphate solution which also flowed through the aperture *r* into the

[1] Lucas, The Effect of Alcohol on the Excitation, Conduction and Recovery Processes of Nerve. J. Physiol., *46*, p. 470; 1913.

first compartment. When in use in the nerve-muscle chamber described in another paper the portion *fg* of the electrode projected into the deeper portion of the muscle nerve cell. The size and shape of the electrode was such as to conform to and be loosely enveloped by the cross sectional contour of the chamber. There is some advantage in having the electrode transparent.

We have tried such electrodes out, and they meet very well the requirements which we had formulated for our experimental work. They appear to us to be so fit for this as well as for other purposes, that

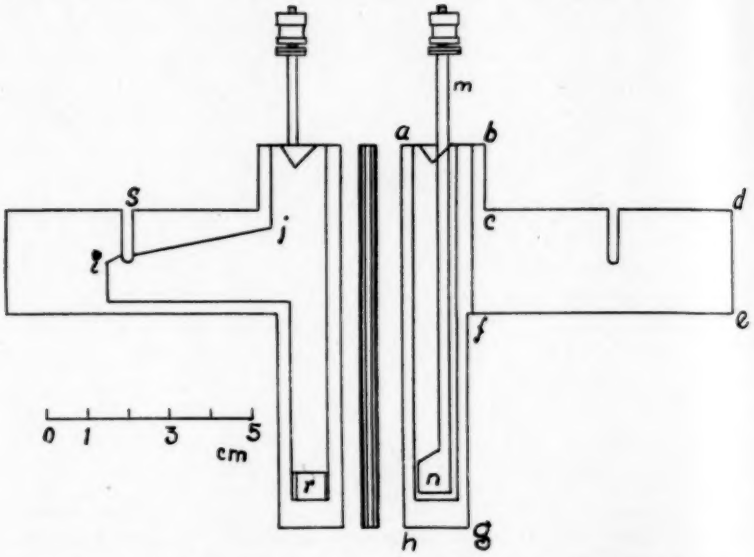

FIG. 1. *Movable non-polarizable electrode.* *S*, Slot to receive nerve; *mn* Zinc Strip.

we believe the description of them may be helpful to others engaged in work where non-polarizable and movable electrodes are used. They may be applied to a nerve without severing the end. The particular size which we have indicated merely chances to fit our present requirements, but they could be made very much smaller for other purposes.

In the development of this electrode a tentative form of it consisted of a celluloid and wood compartment connected by means of a U-tube siphon over the walls of the muscle nerve chamber to the zinc and zinc sulphate chamber. This demanded air-tightness over prolonged intervals and this is not easy to obtain. The filling was very tedious

and air bubbles were very often annoying. Also the siphoning of the fluid which occurred on every disturbance demanded constant attention.

Recently, two other electrodes have been described by Porter[2] and by Pratt,[3] the former for stimulating a nerve, the latter for stimulating small areas of muscle. It has already been stated that our electrode may be applied at any point of a nerve without detaching the nerve at either end. How the electrode may be modified to stimulate small areas of muscle is suggested by Fig. 2. In this case the electrode need

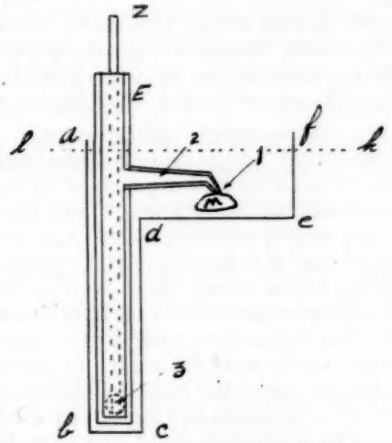

FIG. 2. *The electrode of Fig. 1 designed for stimulating small areas of a muscle*; *a b c d e f*, chamber for muscle *m*. *E*, electrode with small orifice 1 in contact with muscle; *Z*, zinc strip; 3, opening in middle wall allowing communication between the two compartments. Portion 2 may be of rigid construction or may be rubber tube for greater flexibility; *lk*, level of solution.

not be as thin as in that described above. When several electrodes must be stationed side by side along a nerve to stimulate adjacent points along the nerve the width of the electrode must be given attention. For other purposes the electrode may be constructed of glass tubes, or of fibre, rubber or other insulative material. The point in contact with the muscle may be a very small orifice of a capillary tube or an orifice made as Pratt suggests. This capillary portion may be rigidly

[2] Porter, Variations in Irritability of the Reflex Arc. Amer. J. Physiol. *43*, p. 497; 1917.

[3] Pratt, The Excitation of microscopic areas: A Non-polarizable capillary electrode. Amer. J. Physiol., *43*, p. 159; 1917.

attached to the main body of the electrode or may be attached by means of a soft rubber tube for greater flexibility.

We are indebted to Dr. G. N. Stewart for laboratory facilities placed at our disposal, and also to the Cleveland wire division of the National Lamp Works for certain materials.

CUSHING LABORATORY OF EXPERIMENTAL MEDICINE,
WESTERN RESERVE UNIVERSITY,
CLEVELAND, OHIO.
APRIL, 1924.

Radioactivity. —A Report of the Committee on X-rays and Radioactivity of the National Research Council. By A. F. Kovarik and L. W. McKeehan. Bulletin No. 51, March 1925; 203 pages. Published by National Research Council, Washington, D. C., Price $2.25.

This bulletin is a very complete bibliography of papers appearing since 1916 on radioactivity and radioactive measurements. Although direct reference is not made to earlier papers, the ground-work in each particular field is usually discussed briefly. References are also occasionally made to the earlier treatises by Rutherford and Meyer and Schweidler which are used as a natural starting point for this summary. The chapter headings follow closely those used by Rutherford, including, of course, references to the more recent information on these topics.

In their introduction the authors make it clear that no critical selection of papers for discussion has been made. In view of the great mass of material that has accumulated recently this must be regarded as unfortunate. The natural result has been that much valuable space has been given over to work that could well have been given a mere passing reference. Consequently, the authors have been forced to give more important papers a rather hasty discussion. However, their summaries are usually to the point, though brief, and gain somewhat from their conciseness.

The authors have avoided actively engaging in any of the current controversies and have in each case allowed the full claims of the writer of the paper under discussion where these claims have not been challenged in print by others. Although this might mislead one not familiar with the work it does not detract from the value of this summary to persons actively engaged in radioactive work. They will find this compilation of inestimable value in easily and rapidly locating references to the recent work. The literature has been gone over with great thoroughness to June 30, 1924, and their author-index will be found all that could be desired for the period which they aim to cover.

A number of useful tables have been compiled from the papers of this period, including a summary of the γ-ray and β-ray lines, the energies of which have been determined.

L. F. CURTISS

A MUSCLE-NERVE CHAMBER[1]

By H. C. Stevens and Enoch Karrer

In undertaking to repeat the work of Adrian[2] on the all or none principle of the nerve and on certain other characteristics, we have had to develop an apparatus for holding and for stimulating the muscle-nerve preparation. To begin with, we constructed a muscle-nerve chamber out of fibre following the design of Lucas.[3] The Lucas apparatus is shown schematically in Fig. 1. It may be recalled that this muscle-nerve chamber was constructed of a block of ebonite which had three cells *A*, *B*, *C*, chiselled out of it. In the chamber *A* lay the muscle; in chamber *B*, the portion of the nerve to be narcotized, while in

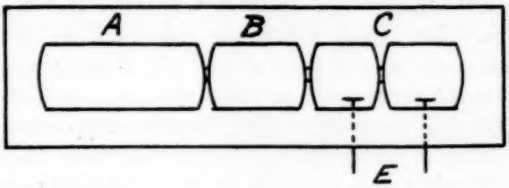

Fig. 1. *The Lucas Chamber.*
A. Muscle cell; *B*, Narcotizing cell; *C*, Stimulating cell, with electrodes at *E* on either side of the stimulating partition.

chamber *C* was the electrode for stimulating the nerve. The cells *A* and *B* and *B* and *C* are connected by a narrow slot in which the nerve is laid. Our general dissatisfaction with the form of this Lucas[3] cell was not so much from any inherent faults as from the particular dimensions and construction of it. The cell is rather small making it possible for the solutions in *A*, *B*, and *C* to become easily contaminated unless they are frequently renewed. The point of stimulation on the nerve did not appear to us to be as definite as it might be and, further-

[1] Most of this work was carried out in the H. K. Cushing Laboratory of Experimental Medicine, Western Reserve University, where equipment and various supplies and services were placed at our disposal. For these we are indebted to Dr. G. N. Stewart, Director, and to Dr. J. M. Rogoff. For the molybdenum sheet and some of the xylophane we are indebted to the Cleveland wire division of the National Lamp Works.

[2] Adrian, E. D. The All-Or-None Principle in nerve. J. Physiol., *47*, p. 460-474; 1913-1914.

[3] Lucas Keith. The Effect of Alcohol on the Excitation, Conduction and Recovery Processes in Nerve. J. Physiol. *46*, p. 470; 1913.

more, this point is fixed. It is of great convenience, and in some of our experiments it was necessary to have the electrodes movable. Another disadvantage was the fact that this cell did not employ non-polarizable electrodes. The electrodes were platinum points inserted through the walls into the cell at opposite sides of the slot in which the nerve lay. The theory here is that the increased concentration of the current in going through the slot around the nerve will stimulate, whereas when the current has emerged on either side of the partition it will spread to the full extent of the cross section of the cell, and therefore, will not be concentrated enough to stimulate the nerve at any other point except at the point where the nerve passes through the partition.

Our first attempt to improve this cell, along the lines which have just been indicated, resulted in the construction of a Lucas cell from a block of wood, in which the partition between the muscle and narcotizing chamber was of molybdenum sheet, and the partition between narcotizing and stimulating chambers was made of two mica sheets while the partition across which the concentrating current would stimulate was made of one mica sheet. All of these partitions were slotted so that the nerve could be laid across them as in the Lucas cell without mechanical strain and without severing the central part of the nerve. The idea in using the molybdenum between the muscle and narcotizing chambers was to eliminate as thoroughly as possible any stray currents that may possibly stimulate the muscle without the intervention of the nerve. The molybdenum partition could be connected to ground, or some other appropriate point in the electrical circuit so that stray currents reaching it would be suitably disposed of. The two mica sheets which constituted one of the partitions that was required to be tight were very close to each other—just allowing another mica sheet without a slot to be pushed between them. The latter small sheet of mica was vaselined and made the partition liquid tight.

We used molybdenum rather than platinum electrodes at opposite sides of the stimulating partition. We found this cell to be satisfactory by actual use, except for the doubt arising from the polarizable nature of the electrodes. Our next step, therefore, was to construct a cell along these lines with the additional improvement of non-polarizable and movable electrodes. It is this cell that we wish to describe in detail here. The description of the non-polarizable and movable electrode is reserved for another paper. Fig. 2 brings out the details of construction of this cell, consisting of the muscle chamber *M* and

narcotizing chamber *N* of variable length, and stimulating chamber *S*. This cell we constructed out of xylophane sheets about 5 mm thick. The particular shape *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* in the horizontal plane was of little concern except that we wished to keep the amount of Ringer's solution to a minimum in the muscle chamber and yet have ample freedom for adjustment of slope and tautness of the filament which connected the muscle to the recording lever.

The walls of the muscle chamber are indicated by *b*, *c*, *d*, *e*, *f*, *g*, *q*, *p*, *o*, Fig. 2, all of which are constructed of xylophane with the exception of

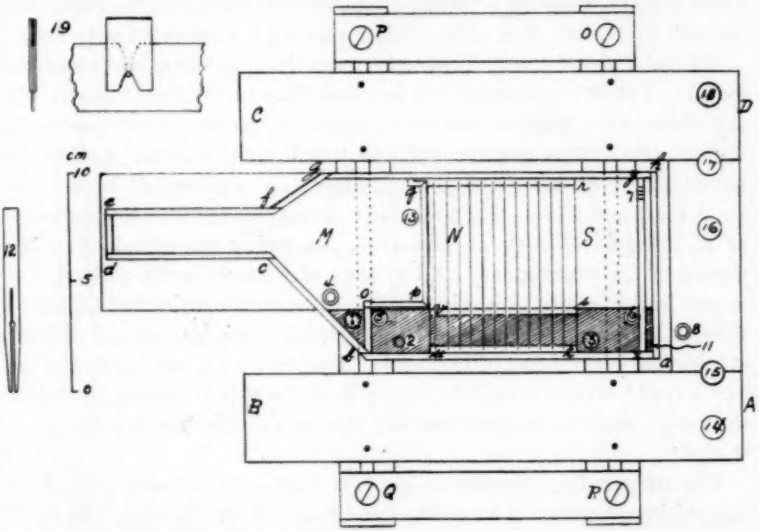

FIG. 2. *Improved Nerve-Muscle Chamber, Top View.*
M, Muscle cell; *N*, narcotizing cell, with grooved walls to allow adjustment of length; *S*, stimulating cell provided with a non-polarizable fixed electrode at right end.

the portion *o*, *p*, *q*. This latter was of the molybdenum sheet as before, about 0.3 mm thick. This molybdenum sheet was bent at right angles along one of its edges and secured to the floor of the cell by means of strips of xylophane and of pins made from ordinary celluloid hair pins or teeth of celluloid combs.

Before continuing with the description, it may not be amiss to point out that this was the method of joining all the walls of the cell and of all other compartments that may be mentioned hereafter. The edges of the xylophane were first made smooth and straight by the use of a fine

file or very fine emery cloth. Holes were then drilled to receive the pins. The pins were always on the slant so that they resisted lateral pressure more securely than if they all had been inserted parallel with each other and perpendicular to the walls of the chamber. After the walls were drilled and fitted, amyl acetate solution of celluloid was spread over the edges and smeared into the holes and then the pins were inserted. After drying for a few hours the pins could be cut off and made flush with the cell walls. We also found that after the cell was constructed we could eliminate all leaks by filling the cell with amyl acetate, allowing to stand for a few moments, and then allowing the cell to dry out thoroughly before putting it in contact with water.

We have already stated that the narcotizing chamber had a variable length. This was accomplished by cementing to the five walls of the cell sheets of Xylophane which had parallel grooves every few millimeters. Into these grooves could be inserted the movable wall of the narcotizing chamber. The grooved sheets are represented by *q r*, *t u*, *v s*, *u v*, *s t*; and *v q r s*. The movable wall may be made of molybdenum or of celluloid as may be preferred, depending somewhat upon the nature of the experiments. All of the partition walls are slotted as is shown in the inserts 19 in Fig. 2. The nerve could easily be laid across these partitions in the grooves without much manipulation and without cutting the central end of the nerve. Rather, to the central end of the nerve could be left attached a portion of the spinal column by which the nerve could be grasped, and which also aided in keeping the nerve straight.

The stimulating chamber *S* extends from the movable wall of the narcotizing chamber to the nearer end wall *XY* of the cell. The small compartment between this near end wall and the outer end wall of the chamber as represented in the figure will be made clear presently.

In cross section our cell resembled an inverted L whose construction and general appearance are depicted in Fig. 3. In the shallow portion of the cell *a h i j* lay the muscle and nerve, while in the deeper portion of the cell *a l k j* was immersed the electrode consisting of zinc sulphate and zinc. The deeper portion is cross-hatched in Fig. 2.

Each of the compartments had an opening in the deep portion for drainage. This is shown in the muscle chamber *M* Fig. 2 at 1; in the narcotizing chamber *N* at 2 and in the stimulating chamber *S* at 3. In each of these chambers we also inserted a glass tube which extended to within about 2 mm of the upper edge of the cell wall and extended below the floor of the cell a centimeter or so. These tubes were designed

for overflow tubes so that in each chamber a continuous flow of fluid can be had if necessary; and at all events they would take care of the accidents sometimes occurring on filling the chambers and on subsequent manipulations. The overflow tubes in the chambers are represented in Fig. 2 by 4, 5, and 6 respectively.

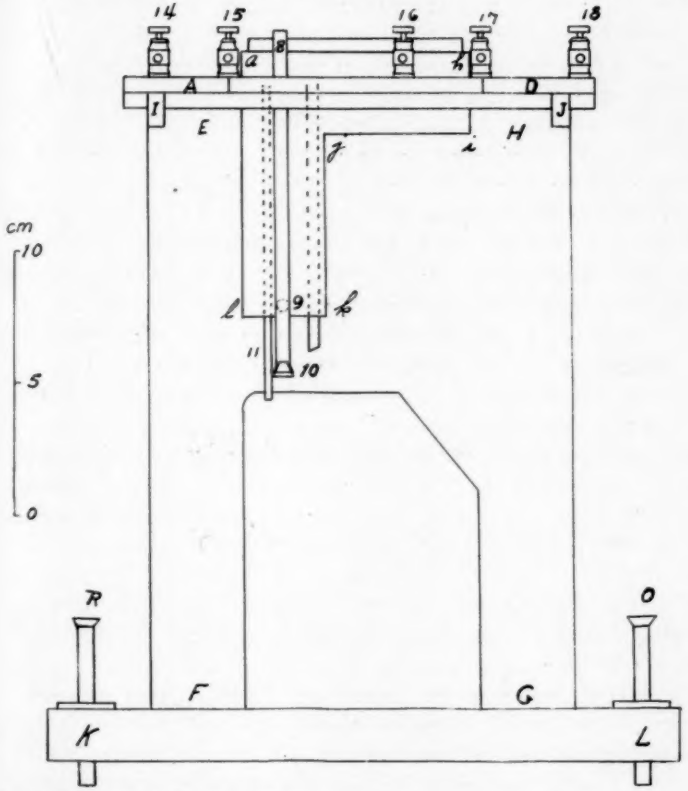

FIG. 3. *Nerve-Muscle Chamber, End View.*
Showing cross-sectional shape that allows the use of movable non-polarizable electrodes.

The end compartment *x y h a*, shown in the top view of Fig. 2, constituted one electrode of our stimulating circuit. This electrode is frequently referred to as the indifferent electrode, since it is at some distance from the nerve and muscle. It consisted of a stationary non-polarizable electrode almost precisely like the electrode which we are

describing in another paper. This electrode compartment communicated with the stimulating chamber *S* by a series of vents at 7 which were on a level with the ends of the overflow tubes 4, 5, 6. A glass tube 8 outside of the chamber communicated with this electrode chamber at 9 (Fig. 3) and was closed at its lower end 10 by means of a small rubber cork. This electrode chamber when in use was filled with zinc sulphate solution for a distance of a few centimeters from the bottom and the Ringer's solution, which filled the stimulating chamber, was allowed to fill the electrode chamber up to the vents 7. Into the glass tube 8 was inserted a strip of zinc until its lower portion dipped into the zinc sulphate solution. At 11, Figs. 2 and 3, is inserted a small glass overflow tube for this electrode chamber.

To support the muscle attached to its bone we used a round wood pin represented in the insert 12 of Fig. 2. The wood pin had a hole just sufficiently large to grip the bone and a slot extending to its lower end, which was filed somewhat conically. This end of the pin was placed into a hole 13 in the floor of the muscle chamber *M*. This mode of holding the muscle was found very satisfactory. The hole 13 extended into but not through a block of xylophane cemented to the under side of the chamber.

As a matter of convenience the cell made of xylophane was mounted upon two thin wood brackets as *E*, *F*, *G*, *H*, of Fig. 3. These brackets were cut out to receive the cell snugly. They are braced by horizontal pieces, *I* and *J* (Fig. 3) at the top end; by the pieces *K* and *L* at the bottom end. Along the upper ends were secured strips of thin wood (*AB*, *CD*, *AD*) for the purpose of holding our movable electrodes and binding posts 14, 15, 16, 17, 18. Leveling screws are shown at *O*, *P*, *Q*, *R* Figs. 2 and 3.

By having a small continuous flow of Ringer's solution into chamber *S* and out by way of the overflow tube 11 one can prevent the diffusion of any of the electrode material (in our case zinc sulphate) to the nerve. Such a flow might also be taken advantage of with our movable stimulating electrode if desired. But this refinement we have not deemed necessary.

The method of making the muscle chamber and narcotizing chamber and stimulating chamber liquid tight was by means of a slotted piece of celluloid shown in inserts 19 Fig. 2. Two pieces of celluloid similarly slotted were cemented together by means of another thin piece of celluloid of just sufficient thickness so that the two pieces may be pushed over the molybdenum mica compartment walls at the slots.

This celluloid saddle was filled with vaseline before pushing it over the slot in the walls. The vaseline was in contact finally with the nerve.

The particular design of our cell is rather immaterial. It may be that where several stimulating electrodes are to be used another deep portion, as cross hatched in Fig. 2, will be of service on the other side of the chambers. We would also point out that the constructions of these cells out of xylophane, which is quite transparent, affords ample opportunity for proper illumination so that all manipulations are made the easier.

SUMMARY

1. The chamber which we have described embodies the essential features of the Lucas chamber in that the complete muscle-nerve preparation is immersed in Ringer's solution and a segment of the nerve may be isolated and subjected to the action of narcotic solutions.

2. The chamber is constructed of transparent material which greatly aids visibility.

3. The size is such as to permit easy manipulation of the preparation and errors due to small quantities of solution are avoided.

4. The system of outlets and overflow tubes permits continuous perfusion and quick change of solutions. This latter in the case of the narcotizing compartment is of great advantage.

5. Stray currents are provided for by the molybdenum partitions and, if desired, by other molybdenum sheets properly placed.

6. The non-polarizable electrode described in another paper, permits symmetrical stimulation of the nerve about its circumference at a point about 2 mm in extent.

7. The non-polarizable electrodes may be moved along the nerve to any desired point.

8. Two or more narcotizing chambers separated by normal nerve may be obtained with this apparatus.

9. The narcotizing chamber may be varied in length from 2 mm to 30 mm.

H. K. CUSHING LABORATORY OF EXPERIMENTAL MEDICINE;
AND WIRE DIVISION NATIONAL LAMP WORKS;
CLEVELAND, OHIO.
APRIL, 1924

Microscopical Examination of Moulding Sand.—The quality of a casting naturally depends upon the condition of the moulding sand. While there are plenty of sand pits in which an entirely satisfactory grade of moulding sand is deposited, many foundries, due to economic

reasons, cannot avail themselves of same, and are compelled to utilize the less satisfactory grade of sand found in pits within their vicinity. In many cases one resorts to mixing of sand of different quality and endeavors in this manner to empirically create a moulding material of as good a quality as existing conditions permit.

Recently a number of papers have been published dwelling upon scientific methods of testing the moulding sand in order to eliminate those less dependable means employed by the moulder in selecting sand, which more or less is a matter of guess work.

The testing methods employed during recent years were based, aside from determining the fire-durability as well as the porosity and permeability of gases, upon chemical analysis. Total analysis, however, was of minor importance and the principal point involved is to establish the ratio of quartz and clay present within the moulding sand, which can be accomplished by means of a rational analysis. The two principal constituents of moulding sand, of which quartz represents the body-substance and clay the agglutinant are distinguished from each other through a definite size of grain and their quantitative relation. For the determination of these two factors several methods have been suggested from time to time. According to "*Schoene-Wahnschaffe*," the size of grain is determined by means of levigation-process, which method has recently been materially improved through suggestions made by "*Treuheit*." According to the latter, the individual grain-fractions can be determined in a reliable and quick manner.

This method of testing is somewhat troublesome and at the same time is a wet process. In its stead one may use a microscope of relatively low power which renders a very satisfactory image of the distribution of the sharply outlined quartz grains in conjunction with the finely powdered clay. An instrument particularly suited to these examinations is the binocular ore-dressing microscope after Schneiderhoehn. Professor Piwowarsky of the Institute of Technology at Aachen (Germany) refers to this apparatus in the journal "Giesserei," Vol. 44, page 721, with the aid of which the predominant grain sizes of new moulding sand and sand already used, as well as of the intermediate products, can be determined within a few minutes.

The circular glass plate, ruled in square cm, fitted within the metal stage of the microscope, facilitates the counting of the individual grains and the determination of their relative quantities. The size of the grains is measured by the aid of net-micrometers of varying values, corresponding to the mesh of sieves as are utilized in dressing. The micrometers referred to are inserted into the ocular tubes. As another eminent advantage it should be mentioned that the binocular ore-dressing microscope yields an image of highly stereoscopic character and that the instrument permits prolonged use without the least eye-strain.

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APPARATUS FOR MAKING ANY LINEAR SCALE

By J. J. Hopfield

INTRODUCTION

In preparing grating spectrograms for rapid comparative study or for publication, it is often desirable to make enlargements of the photographs and to affix to them an accurate, finely spaced scale of wave lengths. There are several ways of doing this, but the usual methods are cumbersome if flexibility and accuracy are desired. The

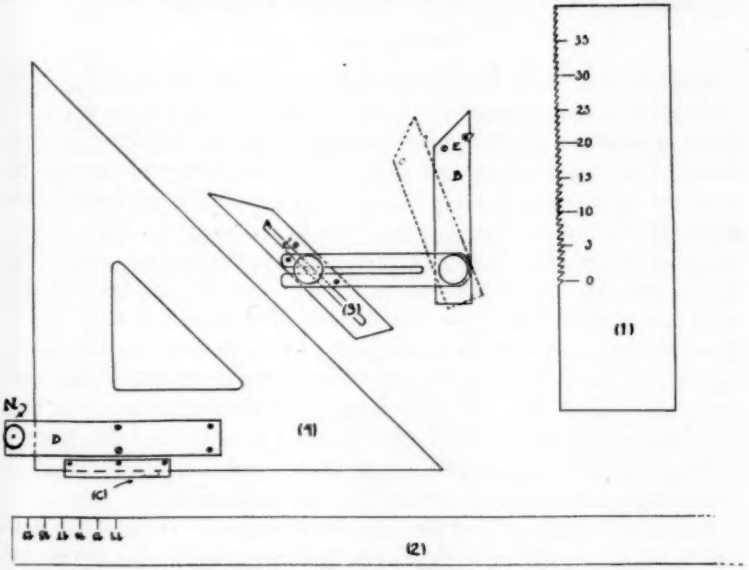

Fig. 1

device described below combines flexibility, accuracy and speed. It is described in the belief that it will be useful not only to spectroscopists but also to draughtsmen and others.

DESCRIPTION OF THE APPARATUS

The apparatus consists of the following pieces: an especially made brass ratchet, (1) Fig. 1; a 50 cm steel scale with mm divisions (2); a

combination bevel (3); and a celluloid right triangle (4). Thus with the exception of the brass rachet, which was made on the milling machine, the apparatus is an assemblage of stock instruments. The necessary modifications of the stock instruments also are represented in Fig. 1 and are as follows: The combination bevel contains a spur on the leg B which meshes into the ratchet. The leg A is firmly fastened to the cross-piece and the bevel is thus limited to one adjustment, that allowed by the hinge on leg B. Finally the legs contain pegs E and E' whose distance apart, y, may be set, as described later, by a vernier caliper so as to fix the angle θ between the legs A and B at any desired value. The triangle (4) has two modifications: namely, a vernier (C) reading to .05 mm's against the millimeter scale (2), and a thin flat spring D, holding a heavy nut N with a prick punch under it.

EXAMPLE OF ITS USE

One can most easily become acquainted with this device by a simple example of its use. Suppose that one wishes to make a linear scale on a piece of paper, disregarding for the time being the length of the divisions. This can be done (see Fig. 1) by translating the triangle (4) down against the steel scale (2) which is fixed with reference to the paper by clamps or thumb tacks, then by bringing the leg A of the combination bevel against the diagonal side of the triangle, and finally by moving the ratchet (1) against the leg B so that the spur meshes with one of the notches, say number 30. The ratchet is now held in place by means of a weight. To make the scale one lifts the nut N against the force of the spring and on releasing it the point pricks the paper and bounds free. Now by moving the bevel downwards so that the spur engages notch 29, another pin prick is made, and the distance between the two points on the paper is the length of a scale division. The above process is repeated for the other scale divisions. The result is a series of equally spaced pin holes on the paper. The division lines are drawn accurately through the pin holes with a pen or sharp pencil. After a little practice the scale may be made with a speed of one division per second.

LENGTH OF A SCALE DIVISION

For the intelligent use of the instrument a knowledge of its geometry is desirable. Fig. 2 represents the exercise described in the preceding paragraph. It is assumed that in this special case the edge of the ratchet was at right angles to the steel ruler, or what amounts to the same thing that the angle θ was 45°. In the dotted square the triangles

(1) and (2) correspond to the positions of the combination bevel and the triangle carrying the pin point respectively, when the point corresponding to 30 was being punched. In order to make the next point the bevel has been moved downwards a distance R equal to the space between the notches 30 and 29 to the position (1′) and the triangle has

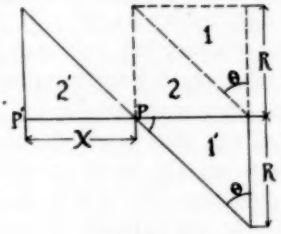

FIG. 2

been pushed to the left in the same operation from P to P', a distance, x, which represents the length of one division of the desired scale. From the figure it is seen that for this case, i.e., for $\theta = 45°$, $x = R$. More will be said about this in a subsequent paragraph. Of course the scale will have equal spaces because the notches are equidistant.

The more general problem of equal spacing is fully represented in Fig. 3. In this figure as in the previous one R, the radius of the circle,

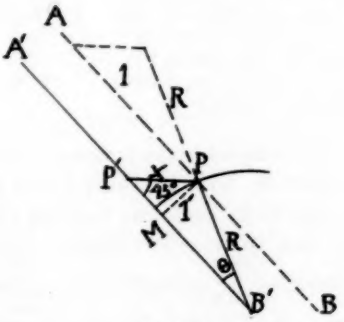

FIG. 3

lies in a direction parallel to the ratchet edge and represents the ratchet spacing. AB and $A'B'$ are lines parallel to the oblique side of the celluloid triangle. The correspondence between this figure and figure 2 is at once evident. The bevel has been moved downwards parallel to the ratchet, a distance R as before, and the triangle has moved from

P to P' a distance, x. The formula for the space length, x, is easily derived thus: Let PM be the perpendicular from P to $A'B'$, then

$$x = \sqrt{2}PM, \text{ and } PM = R\sin\theta; \text{ therefore } x = \sqrt{2}\,R\sin\theta \qquad (1)$$

It follows from this expression that the length of a scale division may be varied continuously between the minimum value $x=0$ for $\theta=0$, and the maximum value $x=\sqrt{2}R$ for $\theta=90°$. If longer spaces than this maximum are desired one may use alternate notches of the ratchet. The interesting special case already mentioned, namely, $x=R$, for $\theta=45°$ follows at once from equation (1).

It also follows from equation (1) that in order to get a desired spacing, x, one needs to set the bevel at the corresponding angle θ. In this case it is not convenient to set to a given angle with accuracy. However, since, y, the distance between the pegs E and E' in figure I, is a function of θ, y as measured by a vernier caliper may be used instead of θ for determining x. This is done by plotting a calibration curve with x and y as coordinates.

A representative problem will now be outlined. The task will be that of actually placing a scale of wave lengths on a spectrogram by interpolation between fiducial spectrum lines. The following data taken from actual spectrograms is taken for example:

I	II	$\Delta\lambda$	ΔS
2304.0A	14.48 cm	305.2	9.16
1998.8	5.32		

Column I gives the fiducial wave lengths. Column II gives the position of these spectrum lines by setting the pin point under N (Fig. 1) above the spectrum lines and reading the vernier C. The meaning of the last two columns is evident. The length of a scale division of 10A is thus $10\times\frac{\Delta S}{\Delta\lambda}$ or 0.3001 cm. The calibration curve (not given) indicates that 8.88 cm is the correct vernier caliper setting for y. The bevel is set accordingly and the length of the scale division is thus fixed. But in the present case neither of the chosen wave lengths is a multiple of 10A. Therefore in order to place the wave length scale correctly one calculates the position of a wave length near one of the fiducial lines which is a multiple of 10 and uses this as a starting point for his scale. In the example chosen such a line is λ2300 and its calcu-

lated scale position is 14.36 cm. The wave length scale may now be made rapidly on the spectrogram. It is placed with the accuracy of the initial vernier settings.

One recognizes that the linear spacing of this device depends on the linear spacing of the notches. If a parabolic scale, a reciprocal scale, a scale of wave lengths for glass prism spectrograms, or any scale whatsoever is desired, it could be made in like manner by having a ratchet with properly spaced notches. However, all except the linear scale have limitations in practice and especially spaced ratchets for other scales would in general not be worth while. Of course with this device any of the above scales can be made with precision although much more slowly without the use of the ratchet. Thus any one of them can be drawn by calculating the position of each scale division and setting the vernier C at the calculated value.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA,
BERKELEY.

Prevention of Blindness.—A report of the National Committee for the Prevention of Blindness declares that "of the 100,000 blind persons in the United States approximately 15,000 are the industrial blind persons who have lost their sight in the pursuit of industrial occupations, and there is in addition to the totally blind a much larger number of men, women and children whose vision has been so impaired by the eye hazards of industry that they are handicapped throughout life. Both of these classes, the industrial blind and the workers whose vision has been seriously impaired by the hazards of other industrial environment, grow larger each year."

There is hardly an industrial occupation in America, according to the National Committee, which does not add each year to the steadily increasing number of the blind and the near blind. Referring to the experience of just one insurance company which in three and a half years settled claims involving 1049 cases of permanent disability resulting from eye injuries, the report says: "The loss of eighty-two eyes in the presumably safe occupations of merchandising, farming, and textile manufacture is further proof of the statement that serious eye accidents are likely to occur wherever men, women and children are employed. There is no such thing as a really non-hazardous occupation."

The Committee found that although from a national point of view the metal manufacturing industries are the source of the greatest number of serious eye injuries, in Pennsylvania the coal mining industry ranks first as a cause of industrial blindness. In Wisconsin it was found that hand tools constitute by far the greatest single cause of eye injuries, being responsible for 44 per cent. As an example of the important part that eye injuries play in the whole problem of industrial accidents the report cites the experience of a large shipbuilding company in whose dispensary 38 per cent of all injuries treated were eye injuries. More than 4300 eye cases were treated in the dispensary of this one company in one year.

The section of the report dealing with the nature and cause of eye injuries closes with the statement: "Until industry knows exactly where, when, how, and why eye accidents occur it will not be in a position to do justice to the existing opportunities for the elimination of such accidents, much less to develop new means of prevention. The keeping of detailed and accurate records of the nature, causes, and costs of eye injuries on the standard basis developed by the International Association of Industrial Accident Boards and Commission is, therefore, recommended as the first step in any campaign for the prevention of eye accidents in the individual industrial plant or in an entire industry."

Copies of the full report, a volume of 250 pages, may be secured from the headquarters of the Committee for the Prevention of Blindness, 130 East Twenty-Second Street, New York City.

A MODIFIED HARTMANN TEST BASED ON INTERFERENCE*

By I. C. Gardner and A. H. Bennett

ABSTRACT

A method of determining the spherical aberration of a lens is given which requires relatively simple apparatus and which gives directly the retardation of different portions of the wave-front referred to any point along the axis and in terms of the wave length of the light employed. Data are given from which a convenient self-contained apparatus can be built to serve as an attachment on an optical bench of usual design. The method is based upon interference phenomena and the bench does not need to be provided with scale or micrometric measuring devices. With the attachment built as suggested, in addition to its dimensions, the measuring of the photographic plate gives all the data required and the one necessary exposure provides a permanent and complete record of the test.

In the Hartmann test for spherical aberration as commonly described no cognizance is taken of the phenomena lying outside the jurisdiction of geometrical optics. The method of test is as follows: The lens to be tested is at L, Fig. 1, and at D is a diaphragm having a series of holes

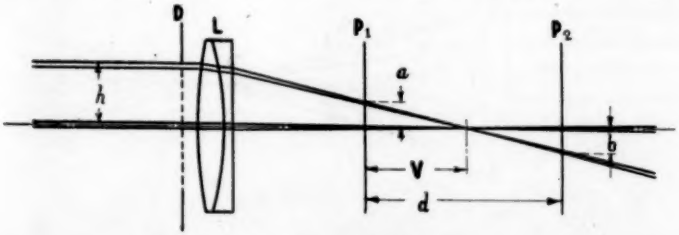

Fig. 1. *The Hartmann Test.*

which may for convenience be equispaced along a diameter of the lens. A distant source or collimated beam of light is employed and two exposures are made in succession upon a photographic plate in the positions P_1 and P_2. On the basis of geometrical optics each small beam transmitted by one of the apertures of the diaphragm will register as a black disc on the plate. For the beam transmitted by any aperture let a and b be the measured distances of the black discs in the two exposures from the optic axis. Then if d, the distance between the positions of the

* Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

plate for the two exposures, is known, it is evident that V, the distance to the intersection of the marginal beam and optic axis measured from plate position P_1, is given by the equation

$$V = \frac{ad}{a+b} \tag{1}$$

and from these values for the different apertures the longitudinal aberration as a function of the incident height h may be determined.

A modified method of making the test will be described which was developed as an outgrowth of the Hartmann test and which resembles in some details the test described by Michelson[1] and later modified by Cotton[2] and Merland.[3] No apparatus is required other than that ordinarily available for the Hartmann test and the spherical aberration is determined directly as the retardation of the different portions of transmitted wave-front in terms of wave length. This is of considerable advantage as it permits the result to be compared with the tolerance for spherical aberration as expressed by Rayleigh. In this test the diaphragm D which is of the Hartmann type is placed behind the lens instead of between the lens and source. See Fig. 2. Only one exposure is

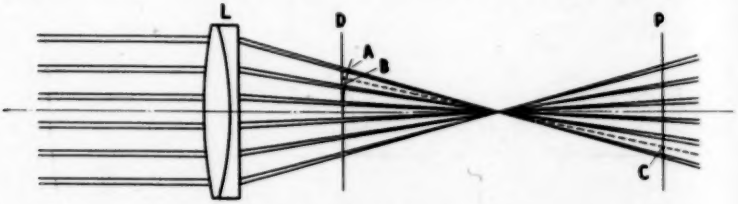

FIG. 2. *The Modified Hartmann test.*

made, with the plate at P. In general, the small beams of the Hartmann test do not register upon the plate as the simple circular discs predicted by geometrical optics but as discs surrounded by diffraction rings.

It will be assumed that the holes in the diaphragm are equispaced and that the plate is placed at such a distance from the focal point that the two first diffraction rings formed by any pair of adjacent apertures overlap. Such an exposure is reproduced without enlargement in Fig. 3. In this case the region of the photographic plate lying between any two adjacent geometrical images receives illumination of approximately

[1] Michelson, Astrophys. J. *47*, p. 283, 1918.
[2] Cotton, Physica *1*, p. 274; 1921.
[3] Merland, Rev. d'Optique, *3*, p. 401; 1924.

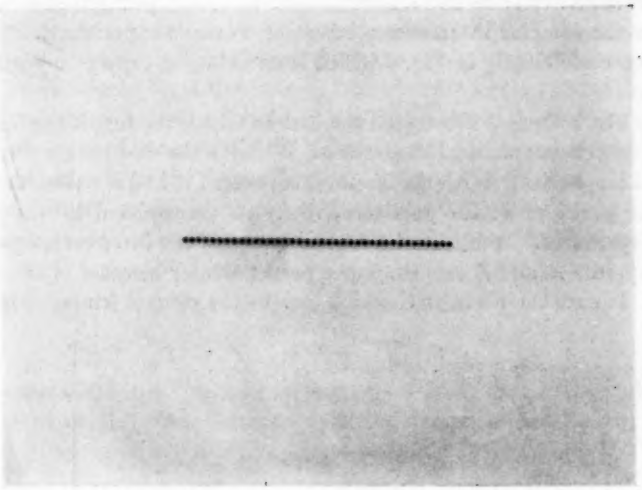

FIG. 3. *Image on plate, diaphragm with circular apertures. Natural size.*

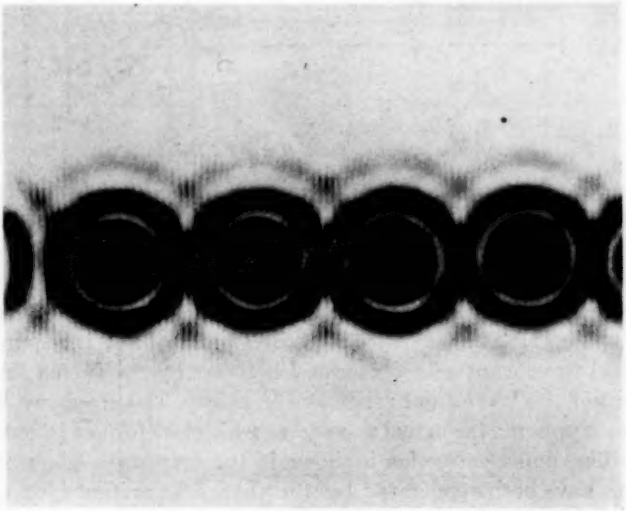

FIG. 4. *Portion of plate shown in Fig. 3 enlarged.*

equal intensity from two adjacent holes of the diaphragm and one obtains the familiar interference bands of Young's experiment. This is shown more clearly in Fig. 4 which is an enlarged copy of a portion of the plate of Fig. 3.

In Fig. 2 there is illustrated the case in which the test is applied to a lens free from spherical aberration. Without the diaphragm the wave front is spherical. With the diaphragm present A and B represent equal phase positions of the partial wave fronts transmitted by the upper two apertures. To locate the central image of the interference pattern join points A and B and draw the perpendicular bisector of the chord AB. It cuts the plate at C which locates the central fringe. This can

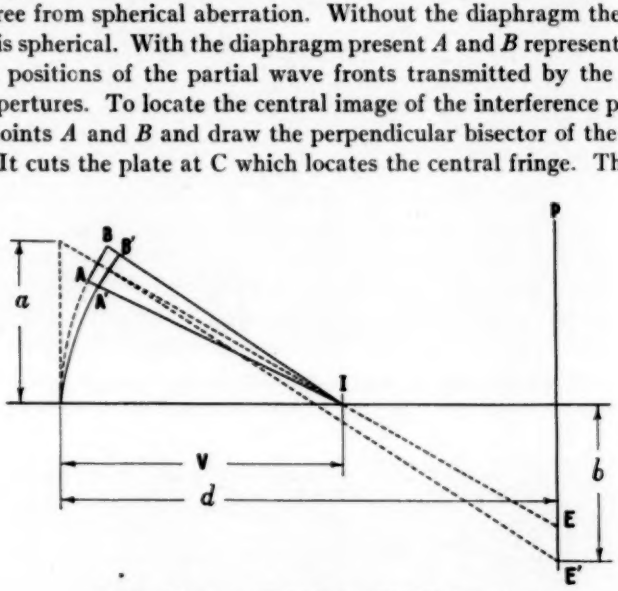

FIG. 5. *Diagrammatic sketch of modified Hartmann test.*

be done in turn for each pair of apertures. But since the wave front is spherical all the bisectors of the chords pass through the center of curvature of the wave front and the central fringes of the several interference patterns will be equispaced on the plate.

For the case in which spherical aberration is present reference is made to Fig. 5. The difference in phase on arrival at I is required. A spherical wave front with center at I is shown by the dotted arc. All portions of this wave front arrive at I in phase. The arc shown by the full line represents the actual wave front which is deformed by spherical aberration (under-correction is shown in the drawing). The marginal portions have been accelerated and the phase acceleration upon arrival at I of any point of the given wave front with respect to the ideal spherical wave is measured by the segment of the radius lying between the two wave fronts. Let points A and B represent two portions of the

spherical wave front transmitted by two adjacent apertures of the diaphragm. Let A' and B' be the corresponding points of the deformed wave front. For the spherical wave front the two sources, as has already been noted, would form the central fringe which lies on the diameter passing midway between A and B. With the actual deformed wave front the central fringe which is measured on the photographic plate lies at E' which is on the perpendicular bisector of the chord $A'B'$. Since the elementary waves from A and B arrive at I in phase, the phase difference on arrival at I for points A' and B' measured in wave lengths is

$$\frac{AA'-BB'}{\lambda} \tag{2}$$

If w is the width of a fringe as formed on the plate and EE' is the displacement of the central fringe resulting from spherical aberration

$$\frac{AA'-BB'}{\lambda} = \frac{EE'}{w} \tag{3}$$

Accordingly, the phase differences are determined if EE' is known for each aperture.

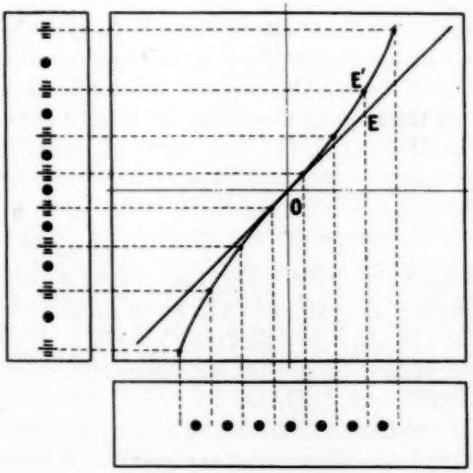

FIG. 6. *Graph for interpreting phase residuals.*

A graphical method for determining EE' will be given from which an analytic solution can be derived. Plot as abscissas (see Fig. 6) the dis-

tances from center of diaphragm (supposedly at optic axis of lens when the exposure was made) to the mid-points between each pair of adjacent holes of the diaphragm. As ordinates plot the distances to the corresponding central fringes. The method of plotting is made clear by the diagrammatic sketches of diaphragm and image on plate below and to the left of graph in Fig. 6. One obtains a graph similar to that shown with a point of inflection at the origin O. Any straight line OE drawn through the origin is the corresponding graph of the equispaced fringes produced by a spherical wave. Straight lines of different slopes correspond to spherical waves having centers at different points on the axis. The straight line OE tangent at the origin to the curve OE' is the graph of the fringes corresponding to a spherical wave with center at the paraxial focus of the lens. Assume that the line is so drawn. The difference in ordinates at two corresponding points of the curve as indicated by the segment EE' is the displacement of the fringe desired and by equation 3 one easily obtains the phase difference for two adjacent apertures. To find the difference of phase between the wave train which travels along the optic axis and that corresponding to any marginal aperture on arrival at the selected axial point it is only necessary to add the differences of phase for all pairs of apertures from the center to the one in question.

If the slope of the straight line in Fig. 6 is changed the differences of phase will vary since changing the slope corresponds to the selection of different points on the optic axis for which the difference of phase is to be obtained. If the slope of the straight line is 1 the point selected is such that (Fig. 5)

$$\frac{d-v}{v}=1 \tag{4}$$

The point of least phase difference, i.e., of best focus, generally lies nearer the lens than the paraxial focus and therefore corresponds to a line having a greater slope than that shown in Fig. 6. It should be noted that the point of best focus does not necessarily correspond to the straight line which most closely fits the curve. This is obvious when it is recalled that one must add up all the intermediate phase differences to get the total phase difference at any point with respect to the ray travelling along the optic axis. A little experience enables one to select the line giving the least phase difference without great difficulty.

The following are the data for the exposure shown in Figs. 3 and 4:

Diameter of apertures in Hartmann diaphragm	1.0 mm
Distance between centers of apertures	2.961 mm
Distance between geometrical images on plate	1.096 mm
Width of fringes on photographic plate	0.140 mm
Distance from diaphragm to focus of lens	519.5 mm
Distance between diaphragm and plate	711.8 mm
Maximum of transmission of filter used at	580 $\mu\mu$

If the positions of the holes in the diaphragm and of the central fringes on the photographic plate are measured with an error not greater than 3μ, an accuracy attained without great difficulty, the corresponding error in phase determination for any pair of holes is approximately $.02\lambda$. This diaphragm was selected as one which had been originally designed for use in the usual Hartmann test.

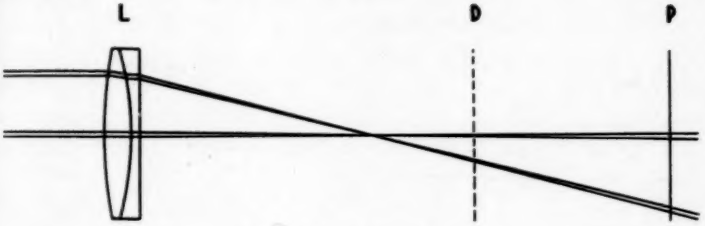

FIG. 7. *Second form of test.*

The use of the above described diaphragm is disadvantageous because of the great distance from diaphragm to focus. A lens having a focal length less than 500 mm cannot be tested. Furthermore, the great distance between diaphragm and plate makes the construction of a large cumbersome apparatus necessary in order to exclude the stray light. An obvious variation which has great advantages is shown in Fig. 7. The diaphragm and photographic plate are placed behind the focus. Instead of circular holes in the diaphragm slits are used. A sample exposure together with an enlarged view of a section of the photograph is shown in Figs. 8 and 9. Fig. 8 is natural size. With this diaphragm the dimensions are as follows:

Width of slits in diaphragm	0.163 mm
Length of slits in diaphragm	3.0 mm
Distance between centers of slits	0.815 mm
Distance between geometrical images on plate	1.278 mm
Width of fringes on photographic plate	0.085 mm

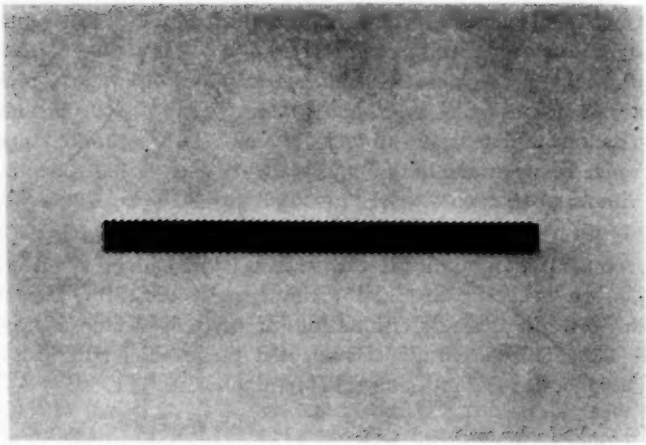

FIG. 8. *Image on plate, diaphragm with parallel slits. Natural size.*

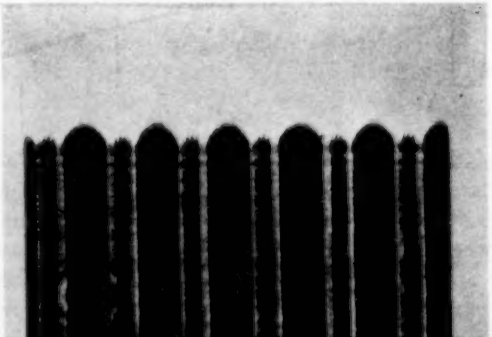

FIG. 9. *Portion of plate shown in Fig. 8 enlarged.*

Distance from diaphragm to focus of lens	244.5	mm
Distance between diaphragm and plate	139.0	mm
Maximum of transmission of filter used at	500	$\mu\mu$

With the diaphragm described above placed behind the focus the test has been applied to an F:4.5 photographic anastigmat. The phase differences have been computed for two axial points, the one lying near the paraxial focus, the second at the point on the axis where it is intersected by a ray proceeding from a point of incidence midway between center and extreme edge of aperture. The table below gives in the first column the height of incidence of ray divided by focal length

TABLE 1.

Height of incident ray in terms of focal length	Phase difference referred to paraxial focus. λ	Phase difference referred to focus for ray incident midway between center and edge of lens. λ
0.011	0.00	+0.01
0.016	−0.01	+0.01
0.022	−0.01	+0.01
0.027	−0.03	+0.02
0.033	−0.04	+0.02
0.038	−0.06	+0.03
0.044	−0.08	+0.03
0.049	−0.12	+0.03
0.054	−0.16	+0.03
0.059	−0.19	+0.03
0.065	−0.22	+0.04
0.070	−0.26	+0.05
0.076	−0.32	+0.04
0.081	−0.38	+0.03
0.087	−0.46	+0.01
0.092	−0.56	−0.03
0.098	−0.67	−0.08
0.104	−0.83	−0.18

of lens. The second column gives the phase difference for the point near the paraxial focus. The third column gives the phase difference for the second point located above. A comparison of the two columns shows that this latter point is much nearer the plane of best imagery than is the paraxial focus. It should be noted that the phase difference tabulated in each case is the total phase difference from center to given ray and is not the partial phase difference between two adjacent apertures. A positive value indicates an acceleration of the wave front relative to its axial portion. In this example, the lens is over-corrected for spherical aberration. The results are shown graphically in Fig. 10.

As the diaphragm is back of the focus as shown in Fig. 7 there are no mechanical difficulties even when a lens of the shortest focal length is to be tested. With an aperture of F:5 the cone of light fills all the apertures of the diaphragm used in preceding test and 60 points are registered on the photographic plate. This is more than are generally required and it is sufficient to measure only every third image when working on a lens of such large aperture. By having so many slits, however, the same diaphragm can be used for testing telescope objectives, in which case an F:15 objective registers ten points on each side

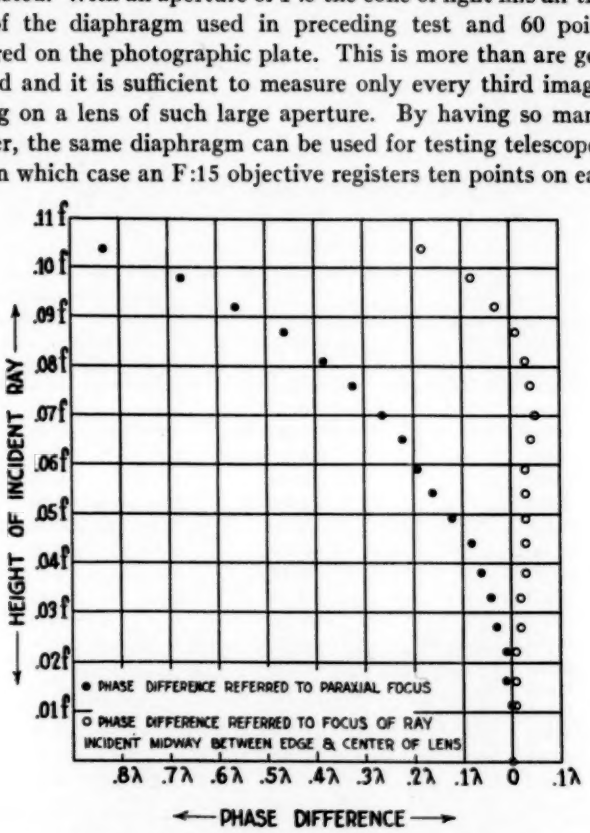

FIG. 10. *Graph showing phase differences for photographic anastigmat.*

of the center. As a source of light a small circular hole with the image of an arc focussed upon it was employed.

It is advantageous to have the diaphragm and the ways to receive the plate holder mounted at the ends of a tube which permits the proper spacing to be secured. One then has a very convenient self contained attachment for use on the optical bench in measuring aberrations. If one also has on the axial line of the instrument a simple finder by which the focus of the lens under test may always be brought into the same

position relative to the modified Hartmann diaphragm each slit of the diaphragm will always correspond to a zone of definite aperture value of the lens. Consequently, the bench need not be provided with scales and all lengths which must be known are either measured directly on the photographic plate or are constants of the testing instrument. The advantages of having a permanent photographic record of the observations made on a lens are evident. The diaphragm can be constructed on an ordinary milling machine with a small circular saw mounted on the spindle to cut the slits. The spacing of the apertures obtained by this method will not be sufficiently uniform to permit irregularities to

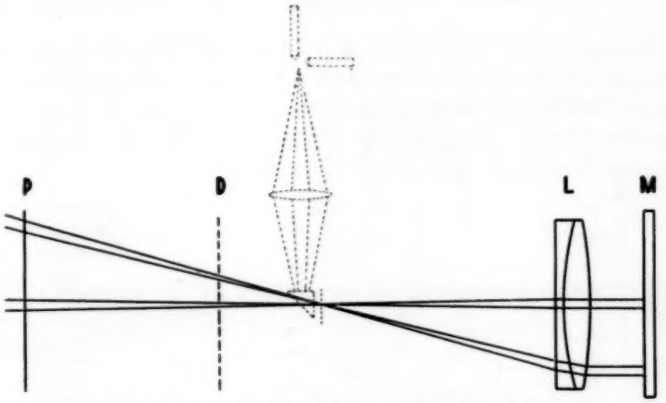

FIG. 11. *Second form of test with autocollimation.*

be ignored. The distance between points midway between successive pairs of apertures are carefully determined and these values are used as abscissas in the diagram of Fig. 6. The results will then not be influenced by the non-uniform spacing.

Placing the diaphragm back of the focus was first tried to enable lenses of short focus to be tested. But, it also has an advantage for lenses of extremely long focus as it makes the application of an autocollimating method convenient. With a telescope lens of long focus a sufficiently distant target cannot be obtained within the laboratory and with the usual Hartmann test an autocollimating method is impracticable. As shown in Fig. 11, however, there is no difficulty when the diaphragm is placed back of the focus. The source, prism and circular aperture are represented by dotted lines. In the diagram they are to be considered as removed from the plane of the paper sufficiently to

permit the returning rays to pass without obstruction. A silvered optical flat is at M.

In designing a diaphragm for use in this method of lens testing there are certain conditions that must be met if satisfactory interference patterns are to be secured. The fringe system is superposed upon the first diffraction band of a single aperture. The angle at the diaphragm subtended by the first diffraction band is λ/s where s is the width of the slit. The angular width of one of the interference bands is λ/e where e is the distance between the slits. Consequently, if one wishes each interference system to consist of n bands

$$e = ns \tag{5}$$

But the diffraction band fades to zero at each edge and experience shows that for normal exposure the two extreme bands are likely to be too faint for use in making the measurements. Therefore, if one wishes three useful bands it is well to set $n = 5$. The width of a fringe on the photographic plate is given by the equation

$$w = \frac{\lambda}{e} d \tag{6}$$

and the distance between geometrical images by the equation[4]

$$e' = \frac{3\lambda d}{S} = 3wn \tag{7}$$

Equation (1) becomes

$$V = \frac{ad}{b \pm a} \tag{8}$$

where the positive sign is to be used if the diaphragm is within the focus and the negative sign if it is placed beyond the focus.

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[4] If circular holes instead of slits are used the distances between geometrical images $e' = \frac{3.24\ \lambda d}{S} = 3.24\ wn$.

AN ELECTRICALLY CONTROLLED MICROMETER CALIPER*

By Charles Moon

I. GENERAL CHARACTER AND PURPOSE OF THE INSTRUMENT

A special micrometer caliper has been developed which is fitted with electrical control for remote operation. The essential parts of the instrument are a headstock and a tailstock supported by a heavy ring frame. The remote control is made possible by driving the headstock screw

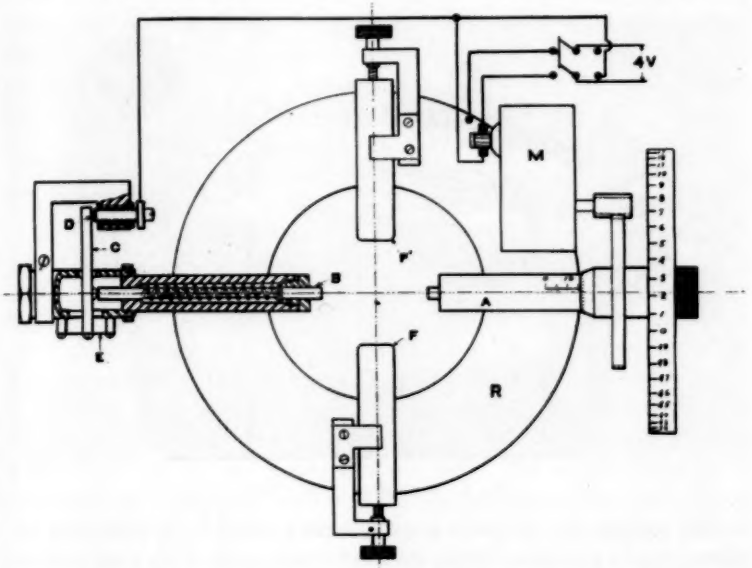

Fig. 1. *Sketch of micrometer. The diameter of the ring is reduced in size relative to the other parts of the apparatus. A micrometer head; B anvil; C contact lever; D contacts; E contact lever spring; FF' guides; M motor and gearing.*

with a very small electric motor, and having in the tailstock a mechanism which automatically stops the motor by opening a set of electric contacts when the critical measuring pressure is reached.

* Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

Hartmann[1] devised an automatic comparator using a screw which was driven by the action of a weight suspended from a cord passing around a pulley. The screw stopped when the frictional forces were increased by pressure to such an extent that they balanced the torque due to the suspended weight. Pratt and Whitney[2] and Shaw[3] have used electrical contacts as indicators in precision measuring machines. The above instruments have had little influence on the present design.

II. DETAILS OF DESIGN

The construction of the instrument is so simple and so well shown by the sketch Fig. 1, and the photograph Fig. 2, that only a brief description of the important parts is given.

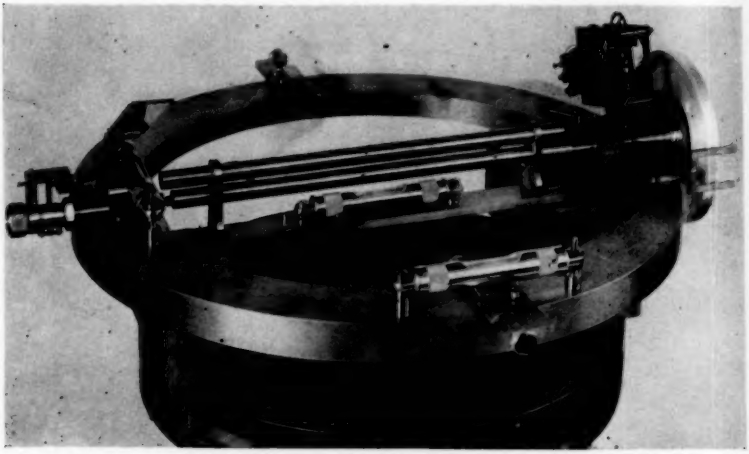

FIG. 2. *Photograph of micrometer.*

The caliper was designed for the special purpose of measuring the diameter of a precision single layer inductance coil, while both coil and caliper were in a constant temperature enclosure. The indicating mechanism was built lighter than would be desirable for general purposes. This was done to reduce friction and make possible a small but definite measuring pressure.

The ease and rapidity of operation, combined with high accuracy, may make the instrument useful for other work. Some of the advan-

[1] Hartmann. La Nature, January 8, 1898.
[2] Glazebrook. Dict. Applied Physics, 3, p. 369.
[3] Proceedings. Institution Mechanical Engineers, Apr. 1913.

tages are: (a) Thermal effects due to the presence of the operator are avoided; (b) settings are made independent of the operator; (c) the approximate setting does not need to be known in advance as with some hand operated calipers.

Frame.—The frame of the caliper is a heavy iron ring cut from a well-seasoned casting. Great care was taken with the alignment of the V-grooves in which the headstock and tailstock are clamped in order that the two measuring surfaces should be parallel. The ring is equipped with two sensitive levels and two guides, F, F'. The latter are useful when measuring the diameter of a cylinder.

Headstock.—The headstock includes the measuring screw with driving motor and reduction gear. Since a precision screw was not available, a commercial micrometer head[4] was used and the screw errors determined by calibration. An index wheel 15 centimeters in diameter graduated into 500 divisions is fitted to the regular divided sleeve. The pitch of the screw is .5 mm so that each division represents a micron. A glass with a hairline ruling is mounted over the graduations so that the readings are readily estimated to 0.1 micron.

The motor which drives the screw has a permanent field and an armature which is a miniature of the three-pole type used extensively in toys. The diameter of the rotor is 1.3 cms and the weight 6.2 grams. The speed of the motor is from 2000 to 8000 revolutions per minute, depending on the voltage applied. The commutator segments are of coin silver with carbon brushes. Between the motor and the screw is a gear train from a small clock which gives a speed reduction of 30000 to 1.

Tailstock.—The tailstock contains the mechanism for controlling the measuring pressure and the automatic arrangement for stopping the motor when the critical pressure is reached. The tailstock plunger is supported on a spiral spring (see B Fig. 1). When making a setting, as the critical pressure is approached, the back end of the plunger presses against the steel ball set into the contact lever at C. A slight further motion opens the platinum contacts at D. The contact lever is so proportioned that the separation of the contacts is about three times as great as the displacement of the steel ball. The measuring pressure is determined by the spiral spring and to some extent by the flat spring on the bottom of the contact lever.

The electric circuit is wired so that the contacts are not in the motor circuit when the switch is thrown to back off the screw. A resistance

[4] By the J. T. Slocomb Co., Providence, R. I.

with short circuiting key is included in the battery circuit. With the key closed, the speed of the motor is about 8000 r.p.m. and open about 2000 r.p.m. The high speed is useful in backing off the screw and on approaching a setting.

III. OPERATION AND TYPICAL RESULTS

The caliper is used only as a comparator to determine the difference in length between an end standard and a desired unknown dimension. The object to be measured is placed between the jaws of the caliper and the switch closed to advance the screw. As already described, the motor is stopped automatically when the pressure reaches a given value. After the contacts open, the inertia of the moving parts carries the screw forward a small distance, which is largely determined by the motor

TABLE 1. *Data showing thirty-six consecutive settings on an end standard made in a period of half an hour*

Reading	Microns	Reading	Microns
1	291859.65	19	291859.59
2	.65	20	.59
3	.65	21	.57
4	.64	22	.57
5	.65	23	.53
6	.65	24	.57
7	.62	25	.56
8	.62	26	.56
9	.64	27	.56
10	.62	28	.54
11	.62	29	.53
12	.62	30	.53
13	.62	31	.53
14	.62	32	.54
15	.62	33	.56
16	.62	34	.53
17	.61	35	.52
18	.58	36	.51

speed. This *over-travel* is normally greater for high speeds of the motor than for low speeds. A peculiar exception to this has been observed when the screw is backed off just enough to bring the contacts together yet not enough to allow the motor to reach full speed before they are again opened. In this case the over-travel of the screw is approximately one-half micron greater than when the circuit is opened at normal full speed. This phenomenon is thought to be due to the oil film on the screw because it nearly disappears when a light watch oil is used. When the

screw is backed off 5 microns or more, the over-travel is so nearly constant that errors cannot be definitely traced to it. The precision with which the automatic stop repeats under favorable conditions is shown by the following data of 36 consecutive settings on an end standard. A micrometer microscope was used to read the fractional parts of a division on the index wheel.

There is a gradual decrease in the readings which may be assigned to temperature change. This total drift is, however, only .14 micron in one-half hour. The maximum difference between two successive readings is .04 micron. The data in Table 1 were obtained without moving the end standard from its position between the caliper jaws.

Table 2 shows some comparative measurements of the difference in length of three end standards made over a period of one year.

TABLE 2. *The difference in length of three end standards Nos.* 291, 292, *and* 293. *The numbers of the end standards are their nominal lengths in millimeters.*

Date	Numbers 293–292	Numbers 292–291
Jan. 1924.....	1.0040 mm	.9967 mm
Jan. 2 1925	1.0035	.9959
Jan. 9 1925	1.0030	.9955
Jan. 16 1925	1.0033	.9964
Jan. 31 1925	1.0035	.9951
Feb. 19 1925	1.0030	.9955
Feb. 19 1925	1.0038	.9956
Feb. 19 1925	1.0034	.9953
Mean of above Readings	1.0034	.9957
* Jan. 1924	1.0031	.9959
* Jan. 1925	1.0033	.9962

* These measurements were given by the gage section of the Bureau.

The greatest deviation from the mean in any of these readings is .4 micron. The end standards were ordinary commercial tools, the spherical faces of which were ground but not polished. It was suspected that this roughness caused a part of the variation found in Table 2. To test this, the faces of the end standards were lapped until a fair reflecting surface was obtained though many scratches still remained. The thickness of the layer removed was about 1.5 microns.

Ten measurements were then made of the difference in length which are given in Table 3.

TABLE 3. *Results of ten measurements of the difference in length of two end standards after the spherical ends were polished. Each difference given is the result of a single independent measurement. No observations were rejected.*

Date	Difference	Date	Difference
March 18, 1924	1.0030 mm	March 19, 1924	1.0029 mm
March 18, 1924	1.0030	March 19 1924	1.0031
March 18, 1924	1.0030	March 30, 1924	1.0031
March 19, 1924	1.0029	March 30, 1924	1.0029
March 19, 1924	1.0028	March 30, 1924	1.0029
		Mean	1.0029_8

The greatest deviation from the mean in this set of readings is only 0.14 micron.

The data given in Table 1 and a great number of other measurements taken with the caliper indicate that the automatic system for stopping the screw can be depended upon to repeat consecutive settings with a precision of a few hundredths of a micron, and errors due to it appear to be small compared to other errors in measuring the difference in length of the end standards.

The writer is especially indebted to Dr. H. L. Curtis for suggestions during the design and testing, and to Mr. C. P. Goddard for his painstaking work in the construction of the caliper.

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MARCH 31, 1925 (4)